Math 172 Quiz 6 Review 17 Oct 2016

1. Find C so that the following are probability density functions on the specified region.

(a) $f(x) = \dfrac{C(x+1)}{x^2+1}$ on $[0,1]$.

$$1 = C\int_0^1 \left(\frac{x+1}{x^2+1}\right) dx$$

$$= C\int_0^1 \left(\frac{x}{x^2+1} + \frac{1}{x^2+1}\right) dx$$

$$= C\left(\frac{1}{2}\ln(x^2+1) + \arctan x\right)\Big|_0^1$$

$$= C\left[\frac{1}{2}\ln 2 - \cancel{\frac{1}{2}\ln 1}^{\,0} + \cancel{\arctan 1}^{\,\frac{\pi}{4}} - \cancel{\arctan 0}\right]$$

so

$$C = \frac{1}{\frac{1}{2}\ln 2 + \frac{\pi}{4}} = \frac{4}{\ln 4 + \pi}$$

Expectations / Grading Rubric

1) Did you set your integral $= 1$?
2) Did you split the integral into two pieces?
3) Did you integrate each piece correctly?
4) Did you simplify $\ln 1 = 0$, $\arctan 0 = 0$, and $\arctan 1 = \frac{\pi}{4}$?
5) Did you solve for C?

(b) $f(x) = \dfrac{C}{(x+1)^{3/2}}$ on $[0,\infty)$.

$$1 = C\int_0^\infty \frac{dx}{(x+1)^{3/2}} = C\int_0^\infty (x+1)^{-3/2}\, dx$$

$$= C\cdot\lim_{R\to\infty}\left[-2\,(x+1)^{-1/2}\right]\Big|_0^R$$

$$= C\lim_{R\to\infty}\left[\cancel{\frac{-2}{\sqrt{R+1}}}^{\,0} + \frac{2}{\sqrt{1}}\right]$$

so $C = \dfrac{1}{2}$

1) See above
2) If you wrote out the u-sub, did you change the limits?
3) Did you integrate correctly?
4) Did you use a limit?
5) Did you solve for C?

2. Use the Comparison Test to show the following either converge or diverge.

(a) $\displaystyle\int_3^\infty \frac{x+1}{\sqrt{x^3-1}}\,dx$

For $x \geq 3$,

$$0 \leq \frac{1}{\sqrt{x}} = \frac{x}{\sqrt{x^3}} < \frac{x+1}{\sqrt{x^3-1}} .$$

Since $\displaystyle\int_3^\infty \frac{dx}{\sqrt{x}}$ is a divergent p-integral $(p = \tfrac{1}{2} \leq 1)$, by comparison $\displaystyle\int_3^\infty \frac{x+1}{\sqrt{x^3-1}}\,dx$ also diverges.

(b) $\displaystyle\int_0^4 \frac{1}{\sqrt{x}+x^5}\,dx$

For $x > 0$,

$$0 < \frac{1}{\sqrt{x}} < \frac{1}{\sqrt{x}+x^5} .$$

Since $\displaystyle\int_0^4 \frac{dx}{\sqrt{x}}$ is a convergent p-integral $(p = \tfrac{1}{2} < 1)$, by comparison $\displaystyle\int_0^4 \frac{1}{\sqrt{x}+x^5}\,dx$ also converges.

1) Did you compare functions, <u>not</u> integrals. Did you include checking that the functions are non negative?

2) Did you reference a known p-integral?

3) Did you state a full conclusion, i.e. that an <u>integral</u> converges/diverges.

4) Is your conclusion correct?

5) Is your argument internally consistent?

Solution and a self grading rubric will be posted by Tuesday evening.