Math 172 | Quiz 7 | Name: ______________
28 Oct 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Please indicate **True** or **False**.

   (a) **T** **F** If $a_n \to 0$ as $n \to \infty$, the series $\sum a_n$ diverges.

   (b) **T** **F** If $a_n \to 0$ as $n \to \infty$, the series $\sum a_n$ converges.

2. $\boxed{1}$ Fill in the blanks.

   For $c \neq 0$, the geometric series $\sum_{n=0}^{\infty} cr^n$ converges if $|r| < 1$ and diverges if $|r| \geq 1$.

3. $\boxed{4}$ Find the partial sums S_3, S_4, S_n, and the sum S for the following series.

$$\sum_{n=1}^{\infty} \frac{4}{(2n-1)(2n+3)} \qquad \text{HINT: } \frac{4}{(2n-1)(2n+3)} = \frac{1}{2n-1} - \frac{1}{2n+3}$$

$$S_3 = \left(1 - \cancel{\tfrac{1}{5}}\right) + \left(\tfrac{1}{3} - \tfrac{1}{7}\right) + \left(\cancel{\tfrac{1}{5}} - \tfrac{1}{9}\right) = 1 + \tfrac{1}{3} - \tfrac{1}{7} - \tfrac{1}{9}$$

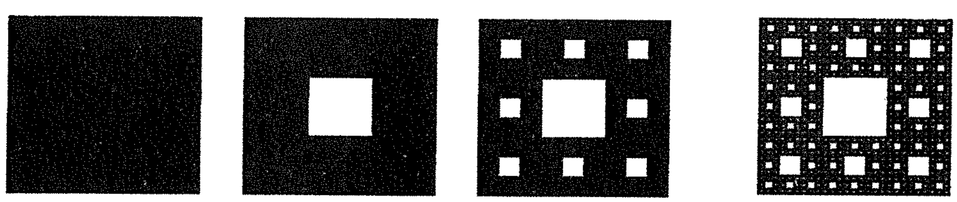
$$S_4 = 1 + \tfrac{1}{3} - \tfrac{1}{9} - \tfrac{1}{11}$$

$$S_n = 1 + \tfrac{1}{3} - \frac{1}{2n+1} - \frac{1}{2n+3} \xrightarrow[n \to \infty]{} \frac{4}{3} = S$$

4. $\boxed{3}$ Let SC_0 be a unit square. Subdivide SC_0 into nine subsquares and remove the middle one, resulting in SC_1. Subdivide the remaining eight subsquares in SC_1 into nine subsquares and remove the middle of each, generating SC_2. The limit of this process is the Sierpiński carpet, SC. The area removed is given by the series

$$\frac{1}{9} + \frac{8}{9^2} + \frac{8^2}{9^3} + \frac{8^3}{9^4} + \frac{8^4}{9^4} + \frac{8^5}{9^6} + \cdots = \frac{1/9}{1 - 8/9} = 1$$

Find the amount of area removed, i.e. the sum of the above series. Make an appropriate series argument.



SC_0 $\quad$ SC_1 $\quad$ SC_2 $\quad$ ... $\quad$ SC