Math 172  
28 Oct 2016  
Quiz 7  
Show Appropriate Work  
Name:  
Point Values in [boxes].

1. **2** Please indicate **True or False**.
   (a) **T**  
      If \( a_n \to 0 \) as \( n \to \infty \), the series \( \sum a_n \) diverges.
   (b) **F**  
      If \( a_n \to 0 \) as \( n \to \infty \), the series \( \sum a_n \) converges.

2. **1** Fill in the blanks.
   For \( c \neq 0 \), the geometric series \( \sum_{n=0}^{\infty} c r^n \) converges if \( |r| < 1 \) and diverges if \( |r| > 1 \).

3. **4** Find the partial sums \( S_3, S_4, S_n \), and the sum \( S \) for the following series.

   \[
   \sum_{n=1}^{\infty} \frac{4}{(2n-1)(2n+3)}
   \]

   **HINT:**
   \[
   \frac{4}{(2n-1)(2n+3)} = \frac{1}{2n-1} - \frac{1}{2n+3}
   \]

   \[
   S_3 = \left( \frac{1}{3} - \frac{1}{5} \right) + \left( \frac{1}{5} - \frac{1}{7} \right) + \left( \frac{1}{7} - \frac{1}{9} \right) + \ldots
   \]

   \[
   S_4 = \left( \frac{1}{3} - \frac{1}{5} - \frac{1}{7} - \frac{1}{9} \right)
   \]

   \[
   S_n = \left( \frac{1}{3} - \frac{1}{2n+1} - \frac{1}{2n+3} \right) \to \frac{4}{3} \quad S
   \]

4. **3** Let \( SC_0 \) be a unit square. Subdivide \( SC_0 \) into nine subsquares and remove the middle one, resulting in \( SC_1 \). Subdivide the remaining eight subsquares in \( SC_1 \) into nine subsquares and remove the middle of each, generating \( SC_2 \). The limit of this process is the Sierpinski carpet, \( SC \). The area removed is given by the series

   \[
   \frac{1}{9} + \frac{8}{9^2} + \frac{8^2}{9^3} + \frac{8^3}{9^4} + \frac{8^4}{9^5} + \frac{8^5}{9^6} + \ldots = \frac{\frac{1}{9}}{1 - \frac{8}{9}} = \frac{1}{9}
   \]

   Find the amount of area removed, i.e. the sum of the above series. Make an appropriate series argument.