1. (a) **Geometric Series.**
   - For \( c \neq 0 \), the geometric series \( \sum_{n=0}^{\infty} c r^n \) converges to \( \frac{c}{1-r} \) for \( |r| < 1 \), and diverges for \( |r| \geq 1 \).

(b) **Test for Divergence.**
   - If \( a_n \not\to 0 \) as \( n \to \infty \), then \( \sum a_n \) diverges.

(c) **P-Series.**
   - \( \sum_{n=1}^{\infty} \frac{1}{n^p} \) converges for \( p > 1 \), and diverges for \( p \leq 1 \).

(d) **Integral Test.**
   - If \( a_n = f(n) \) and \( f(x) \) is positive, continuous & decreasing, then the Integral Test applies.

2. (a) \( \sum_{n=2}^{\infty} \frac{\sqrt{1+n^3}}{n^3+4n} \) compare to \( \sum \frac{n^{7/3}}{n^3} = \sum \frac{1}{n^{1/3}} \) so it **CONVERGES**.

(b) \( \sum_{n=2}^{\infty} \frac{4+2n}{n^2+4n+1} \) compare to \( \sum \frac{1}{n} \) so it **DIVERGES**.

3. **Use the Limit Comparison Test to show the following converges.**

\[ \sum \frac{2n^2+4}{\sqrt{5n^8+4n^2}} \]

Since \( \sum \frac{2n^2+4}{\sqrt{5n^8+4n^2}} \) & \( \sum \frac{1}{n^2} \) are both positive we can apply the L.C.T.

We compute \( \lim_{n \to \infty} \frac{2n^2+4}{\sqrt{5n^8+4n^2}} / \frac{1}{n^2} = \frac{2}{15} \) since \( 0 < \frac{2}{15} < \infty \).

is a convergent \( p \)-series \( (p > 1) \), by the Limit Comparison Test.

\[ \sum \frac{2n^2+4}{\sqrt{5n^8+n^2}} \] also converges.