

1. 4 Complete the following statements.

(a) GEOMETRIC SERIES.

- For  $c \neq 0$ , the geometric series  $\sum_{n=0}^{\infty} cr^n$  converges to  $\frac{c}{1-r}$  for  $|r| < 1$ , and diverges for  $|r| \geq 1$ .

(b) TEST FOR DIVERGENCE.

- If  $a_n \not\rightarrow 0$  as  $n \rightarrow \infty$ , then  $\sum a_n$  diverges.

(c) P-SERIES.

- $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges for  $p > 1$ , and diverges for  $p \leq 1$ .

(d) INTEGRAL TEST.

- If  $a_n = f(n)$  and  $f(x)$  is positive, continuous & decreasing, then the Integral Test applies.

2. 2 For the following series, specify what series you would compare each to and based on your comparison, decide if it converges or diverges. No formal justification is needed.

(a)  $\sum_{n=2}^{\infty} \frac{\sqrt[3]{1+n^7}}{n^3+4n}$  compare to  $\sum \frac{n^{7/3}}{n^3} = \sum \frac{1}{n^{2/3}}$  so it CONVERGES / DIVERGES

(b)  $\sum_{n=2}^{\infty} \frac{4+2n}{n^2+4n+1}$  compare to  $\sum \frac{1}{n}$  so it CONVERGES / DIVERGES

3. 4 Use the Limit Comparison Test to show the following converges.

$$\sum \frac{2n^2+4}{\sqrt{5n^8+4n^2}}$$

Since  $\sum \frac{2n^2+4}{\sqrt{5n^8+4n^2}}$  &  $\sum \frac{1}{n^2}$  are both positive we can apply the L.C.T.

We compute  $\lim_{n \rightarrow \infty} \frac{2n^2+4}{\sqrt{5n^8+4n^2}} \bigg/ \frac{1}{n^2} = \frac{2}{\sqrt{5}}$ . Since  $0 < \frac{2}{\sqrt{5}} < \infty$  &  $\sum \frac{1}{n^2}$

is a convergent p-series ( $p=2 > 1$ ), by the Limit Comparison Test,

$\sum \frac{2n^2+4}{\sqrt{5n^8+4n^2}}$  also converges.