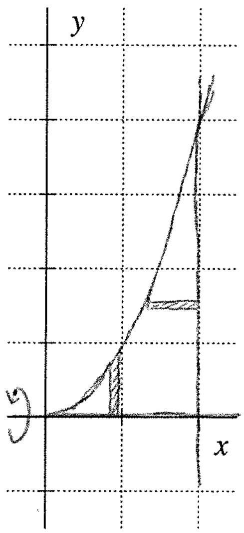
Math 172 Worksheet 1 Name: ______________

14 Sept 2016 Show Appropriate Work Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ The region bounded by the graphs of $y = x^2$, $x = 2$, and $y = 0$ is revolved around the x-axis. Sketch the region on the provided grid. Express the volume of the resulting solid as an integral using the Disk Method and the Shell Method. Evaluate both integrals and verify they agree.

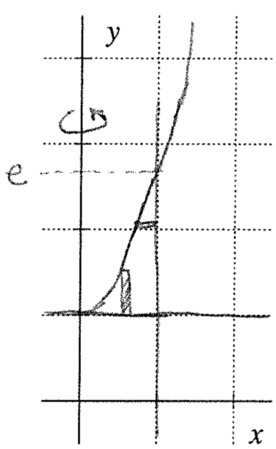


(a) $V_{Disk} = \pi \int_0^2 (x^2)^2\,dx = \pi \frac{x^5}{5}\Big|_0^2 = \frac{32\pi}{5}$

(b) $V_{Shell} = 2\pi \int_0^4 y\,(2-\sqrt{y})\,dy = 2\pi\left(y^2 - \frac{2}{5}y^{5/2}\right)\Big|_0^4$

$= 2\pi\left(16 - \frac{64}{5}\right) = \frac{32\pi}{5}$

2. $\boxed{3}$ The region bounded by the graphs of $y = e^{x^2}$, $x = 1$, and $y = 1$ is revolved around the y-axis. Sketch the region on the provided grid. Express the volume of the resulting solid as an integral using the Disk Method and the Shell Method. **Do not evaluate either integral yet.**



(a) $V_{Disk} = \pi \int_1^e \left[(1)^2 - (\sqrt{\ln y})^2\right] dy = \pi \int_1^e (1 - \ln y)\,dy$

(b) $V_{Shell} = 2\pi \int_0^1 x\,(e^{x^2} - 1)\,dx$

(c) One of the two integrals above can be evaluated using techniques we have discussed this semester. Evaluate that integral.

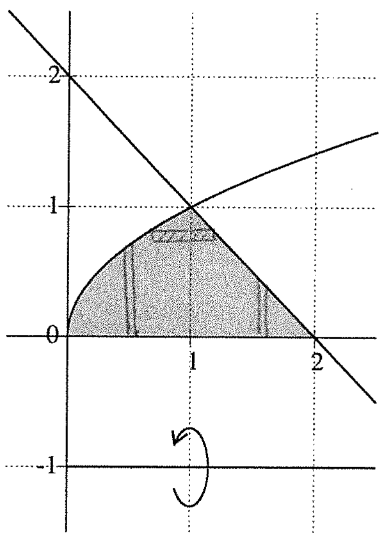
$V_{Shell} = 2\pi \int_0^1 x e^{x^2}\,dx - 2\pi \int_0^1 x\,dx = \pi \int_0^1 e^u\,du - 2\pi \frac{x^2}{2}\Big|_0^1$

$u = x^2$
$du = 2x\,dx$
$0 \mapsto 0$
$1 \mapsto 1$

$= \pi e^u\Big|_0^1 - \pi = \pi(e-1) - \pi$

$= \pi(e-2)$

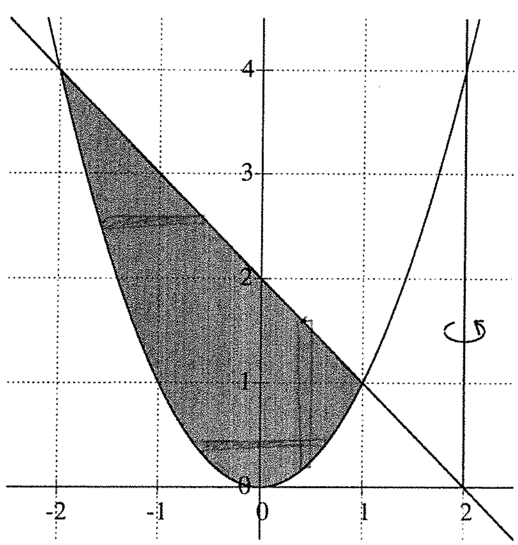
3. $\boxed{2}$ The region bounded by the graphs of $y = \sqrt{x}$, $y = 2 - x$, and the x-axis, the shaded region in the figure, is revolved around the line $y = -1$. Express the volume of the resulting solid as an integral using the Disk Method and the Shell Method. **Do not evaluate either integral.**



(a) $V_{Disk} = \pi \int_0^1 \left((\sqrt{x}+1)^2 - 1^2 \right) dx + \pi \int_1^2 \left((3-x)^2 - (1)^2 \right) dx$

(b) $V_{Shell} = 2\pi \int_0^1 (y+1)(2-y-y^2)\, dy$

4. $\boxed{3}$ The region bounded by the graphs of $y = x^2$ and $y = 2 - x$, the shaded region in the figure, is revolved around the line $x = 2$. Express the volume of the resulting solid as an integral using the Disk Method and the Shell Method. **Do not evaluate either integral.**



(a) $V_{Disk} = \pi \int_0^1 \left((2+\sqrt{y})^2 - (2-\sqrt{y})^2 \right) dy + \pi \int_1^4 \left((2+\sqrt{y})^2 - (y)^2 \right) dy$

(b) $V_{Shell} = 2\pi \int_{-2}^1 (2-x)(2-x-x^2)\, dx$