

Geometric Series:

- A geometric series $\sum_{n=0}^{\infty} cr^n$ with $c \neq 0$ converges to $\frac{c}{1-r}$
if $|r| < 1$ and diverges if $|r| \geq 1$.

Telescoping Series:

- To find the sum of a convergent telescoping series,

1. Find a formula for S_n .

2. Then take the limit as $N \rightarrow \infty$.

The series converges if the limit exists and diverges if the limit does not.

Divergence Test:

- A series $\sum a_n$ diverges if $\lim_{n \rightarrow \infty} a_n \neq 0$.
- We know nothing if $\lim_{n \rightarrow \infty} a_n = 0$.

$$\text{Harmonic Series} = \sum \frac{1}{n}.$$

- The harmonic series converges/diverges (circle one).

$$\text{P-Series: } \sum \frac{1}{n^p}$$

- Converges if $P > 1$.
- Diverges if $P \leq 1$.

Integral Test:

- State the assumptions:

Let $f(n) = a_n$, if $f(x)$ is positive, continuous, and decreasing then

The series $\sum_{n=1}^{\infty} a_n$ converges if $\int_1^{\infty} f(x) dx$ converges and diverges if $\int_1^{\infty} f(x) dx$ diverges.

Direct Comparison Test:

- $\sum a_n$ converges if $0 \leq a_n \leq b_n \quad \text{if } \sum b_n \text{ converges}$.
- $\sum a_n$ diverges if $0 \leq b_n \leq a_n \quad \text{if } \sum b_n \text{ diverges}$.

Limit Comparison Test (LCT):

$$L = \lim_{n \rightarrow \infty} \frac{a_n}{b_n} \quad a_n > 0, \quad b_n > 0$$

- Case 1: $L > 0$, so $\sum a_n \text{ if } \sum b_n \text{ behave the same}$.
- Case 2: $L = 0$, so $\text{if } \sum b_n \text{ converges, } \sum a_n \text{ converges}$.
- Case 3: $L = \infty$, so $\text{if } \sum b_n \text{ diverges, } \sum a_n \text{ diverges}$.

Alternating Series Test (AST):

- The series $\sum (-1)^n b_n$ converges if: $b_n > 0, \quad b_n > b_{n+1}, \quad b_n \xrightarrow{n \rightarrow \infty} 0$
- For convergent alternating series, we know that $|S - \frac{S_N}{b_{N+1}}| \leq b_{N+1}$.

Absolute Convergence:

- The series $\sum |a_n|$ converges absolutely if $\sum |a_n| \text{ converges}$.

Conditional Convergence:

- The series $\sum a_n$ converges conditionally if $\sum |a_n| \text{ diverges but } \sum a_n \text{ converges}$.

Ratio/Root Test:

$$\text{Ratio: } \rho = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$\text{Root: } \rho = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$$

- The series $\sum a_n$ converges if: $|\rho| < 1$.
- The series $\sum a_n$ diverges if: $|\rho| > 1$.
- Test inconclusive if: $|\rho| = 1$.