1. Evaluate.

(a) \[
\int_1^2 \left(1 + \frac{3x^2}{\sqrt{x^3 + 8}}\right) \, dx
\]

\[
= \int_1^2 \frac{3x^2}{\sqrt{x^3 + 8}} \, dx + \int_1^2 \frac{1}{\sqrt{x^3 + 8}} \, dx
\]

\[
= x \bigg|_1^2 - \frac{1}{3} \left( \ln |x^3 + 8| \right) \bigg|_1^2
\]

\[
u = x^3 + 8, \quad du = 3x^2 \, dx
\]

\[
1 \to 9 \quad 2 \to 10
\]

\[
= 1 + 2 \left( 4 - 3 \right) = 3
\]

(b) \[
\int_0^1 \frac{dy}{1 + 3y^2}
\]

\[
u = \sqrt{3} y, \quad dv = \sqrt{3} dy, \quad v = \sqrt{3}
\]

\[
= \frac{1}{\sqrt{3}} \int_0^1 \frac{du}{1 + u^2}
\]

\[
= \frac{1}{\sqrt{3}} \left[ \arctan u \right]_0^1 = \frac{1}{\sqrt{3}} \left( \arctan \sqrt{3} - \arctan 0 \right)
\]

\[
= \frac{1}{\sqrt{3}} \left( \frac{\pi}{3} \right)
\]
2. Consider the region bounded by the graphs of \( y = \frac{x}{2} \) and \( y = \sqrt{x} \).

(a) Carefully sketch the region.

(b) Express the area of the region as an integral with respect to \( x \), i.e., a \( dx \) integral. Do Not Evaluate.

\[
\int_{0}^{1} \left( \sqrt{x} - \frac{x}{2} \right) \, dx
\]

(c) Express the area of the region as an integral with respect to \( y \), i.e., a \( dy \) integral. Do Not Evaluate.

\[
\int_{0}^{2} \left( 2y - y^{2} \right) \, dy
\]

(d) A solid has base given by the region and cross sections perpendicular to the \( y \)-axis are squares. Express the volume of the solid as an integral. Do Not Evaluate.

\[
\int_{0}^{2} \left( 2y - y^{2} \right)^{2} \, dy
\]