

1. Evaluate.

(a) 5

$$\int_1^2 \left(1 + \frac{3x^2}{\sqrt{x^3+8}} \right) dx$$

$$\begin{aligned}
 &= \int_1^2 dx + \int_1^2 \frac{3x^2}{\sqrt{x^3+8}} dx = x \Big|_1^2 + \int_9^{16} u^{-1/2} du \\
 &\quad u = x^3 + 8 \\
 &\quad du = 3x^2 dx \\
 &\quad 1 \mapsto 9 \\
 &\quad 2 \mapsto 16 \\
 &= (2-1) + 2u^{1/2} \Big|_9^{16} \\
 &= 1 + 2(4-3) = 3
 \end{aligned}$$

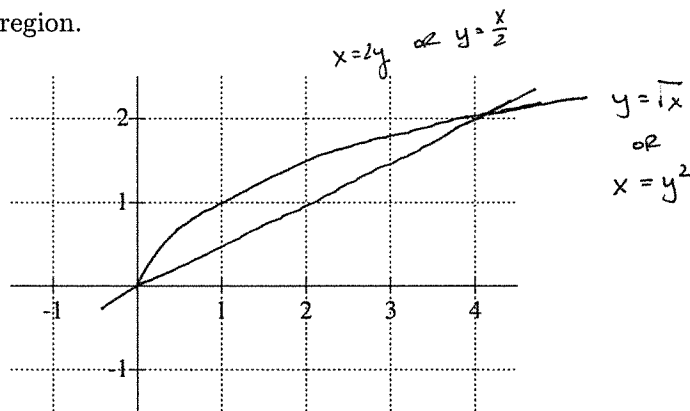
(b) 5

$$\begin{aligned}
 &\int_0^1 \frac{dy}{1+3y^2} \\
 &\quad u = \sqrt{3}y \quad 0 \mapsto 0 \\
 &\quad du = \sqrt{3} dy \quad 1 \mapsto \sqrt{3} \\
 &= \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} \frac{du}{1+u^2} \\
 &= \frac{1}{\sqrt{3}} \arctan u \Big|_0^{\sqrt{3}} = \frac{1}{\sqrt{3}} \left(\arctan \sqrt{3} - \arctan 0 \right) \\
 &= \frac{1}{\sqrt{3}} \left(\frac{\pi}{3} \right)
 \end{aligned}$$

CONTINUED ON THE REVERSE.

2. Consider the region bounded by the graphs of $y = x/2$ and $y = \sqrt{x}$.

(a) 1 Carefully sketch the region.



(b) 3 Express the area of the region as an integral with respect to x , i.e., a dx integral. **Do Not Evaluate.**

$$\int_0^4 \left(\sqrt{x} - \frac{x}{2} \right) dx$$

(c) 3 Express the area of the region as an integral with respect to y , i.e., a dy integral. **Do Not Evaluate.**

$$\int_0^2 (2y - y^2) dy$$

(d) 3 A solid has base given by the region and cross sections perpendicular to the y -axis are squares. Express the volume of the solid as an integral. **Do Not Evaluate.**

$$\int_0^2 (2y - y^2)^2 dy$$