

## Math 172 Quiz 10

Sections: 11.3, 11.4

7 Dec 2018

Name: \_\_\_\_\_ Point values in  boxes.

1. Convert the following polar equations into rectangular coordinates, or rectangular to polar expressing your solution in the form  $r = f(\theta)$ .

(a)  1       $r = \frac{4}{2 \sin \theta - \cos \theta}$

$$2r \sin \theta - r \cos \theta = 4$$

$$2y - x = 4 \quad \text{or} \quad y = \frac{x}{2} + 2$$

(b)  1       $(x+2)^2 + y^2 = 4$

$$x^2 + 4x + 4 + y^2 = 4 \quad r = -4 \cos \theta$$

$$x^2 + y^2 + 4x = 0$$

$$r^2 + 4r \cos \theta = 0$$

2.  3 Find the length of the polar curve  $r = \sec \theta$  for  $\theta \in [0, \pi/4]$ .

$$r = \sec \theta$$

$$r' = \sec \theta \tan \theta$$

$$\begin{aligned} r^2 + (r')^2 &= \sec^2 \theta + \sec^2 \theta \tan^2 \theta \\ &= \sec^2 \theta (1 + \tan^2 \theta) \end{aligned}$$

$$ds = \sec \theta \sqrt{1 + \tan^2 \theta} d\theta$$

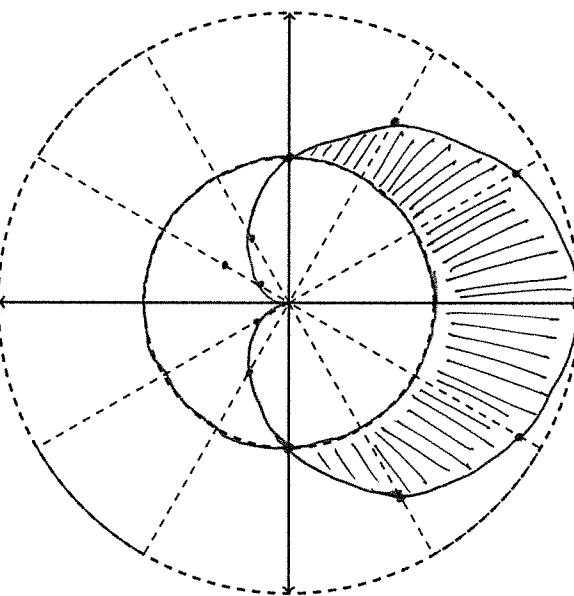
$$\begin{aligned} s &= \int_0^{\pi/4} \sec \theta \sqrt{1 + \tan^2 \theta} d\theta \\ &\quad \text{with } u = \tan \theta \\ &= \int_0^{\pi/4} \sec^2 \theta d\theta \\ &= \left[ \sec \theta \right]_0^{\pi/4} = \tan \theta \Big|_0^{\pi/4} = 1 \end{aligned}$$

CONTINUED ON REVERSE.

$$\text{Given: } \sin(2x) = 2 \sin x \cos x \quad \left| \right| \quad \sin^2 x = (1 - \cos(2x))/2 \quad \left| \right| \quad \cos^2 x = (1 + \cos(2x))/2$$

3. Polar area and graphing.

- (a) [2] Sketch the curves  $r = 1$  and  $r = 1 + \cos \theta$ .



- (b) [3] Find the area inside the curve  $r = 1 + \cos \theta$  but outside the curve  $r = 1$ .  
Shade the region to indicate the area you are trying to find.

$$\begin{aligned}
 A_{\text{area}} &= 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{2}} \left( (1 + \cos \theta)^2 - 1 \right) d\theta \\
 &= 2 \sin \theta + \frac{\theta}{2} + \frac{1}{4} \sin 2\theta \Big|_0^{\frac{\pi}{2}} \\
 &= 2 + \frac{\pi}{4} \\
 &= \int_0^{\frac{\pi}{2}} (2 \cos \theta + \cos^2 \theta) d\theta \\
 &= \int_0^{\frac{\pi}{2}} \left( 2 \cos \theta + \frac{1}{2} (1 + \cos 2\theta) \right) d\theta
 \end{aligned}$$