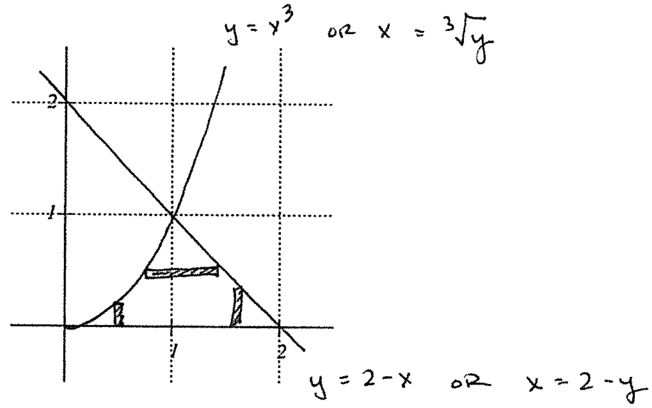
Math 172 Quiz 2
Sections: 6.3-6.5
14 September 2018

Name: ____________________

Point values in $\boxed{\text{boxes}}$.

1. $\boxed{1}$ Carefully sketch the region bounded by the graphs of $y = x^3$ and $y = 2 - x$ and the x-axis.



(a) $\boxed{2}$ The region is rotated about the x-axis. Express the volume using both the Disk Method and the Shell Method. Do not integrate.

$$V_{Disk} = \pi \int_0^1 (x^3)^2\, dx + \pi \int_1^2 (2-x)^2\, dx$$

$$V_{Shell} = 2\pi \int_0^1 y\left((2-y) - \sqrt[3]{y}\right) dy$$

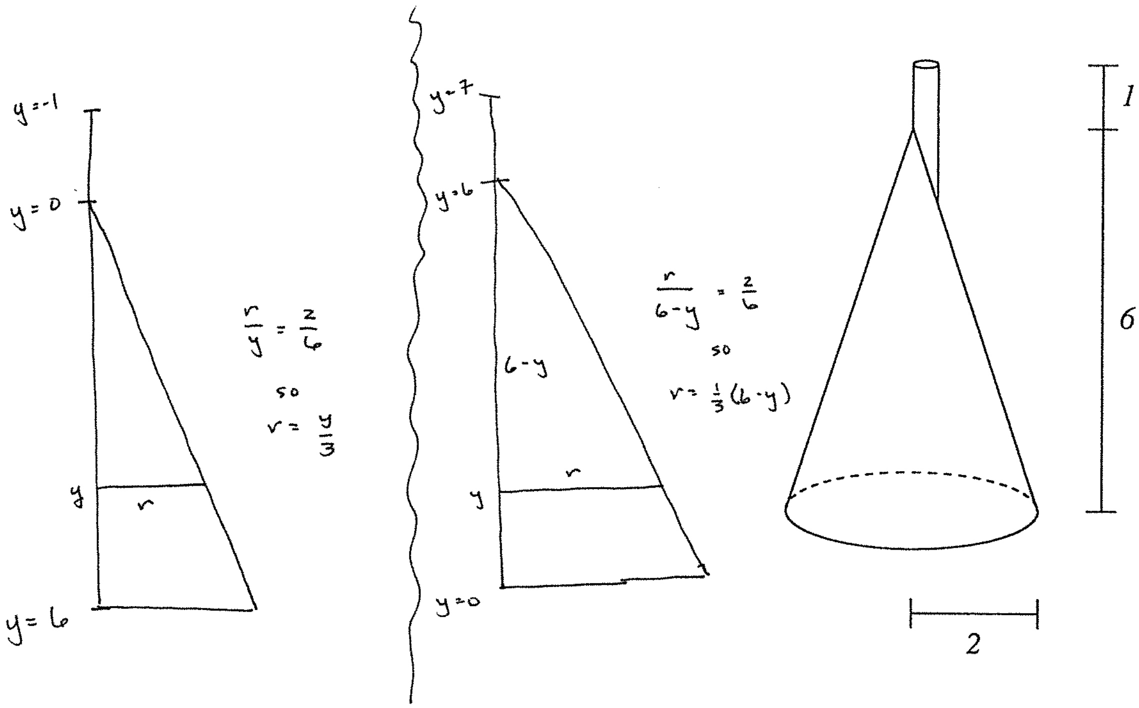
(b) $\boxed{2}$ The region is rotated about the line $x = 2$. Express the volume using both the Disk Method and the Shell Method. Do not integrate.

$$V_{Disk} = \pi \int_0^1 \left[(2 - \sqrt[3]{y})^2 - (2-(2-y))^2\right] dy$$

$$V_{Shell} = 2\pi \int_0^1 (2-x)\, x^3\, dx + 2\pi \int_1^2 (2-x)(2-x)\, dx$$

CONTINUED ON THE REVERSE.

2. A conical tank on the moon is filled with liquid oxygen of density ρ. The gravitational constant on the moon is g. The tank is 6 m tall and has radius 2 m at the base. There is a spout protruding 1 m above the top of the cone. The cone is oriented as shown in the figure below. **Please start by clearly identifying a coordinate system.**



(a) $\boxed{4}$ If the tank is full, express the work required to empty the tank through the spout as an integral. **Do not evaluate the integral.**

$$W = \rho g \pi \int_0^6 \left(\frac{y}{3}\right)^2 (y+1)\, dy$$

$$W = \rho g \pi \int_0^6 \left(\frac{6-y}{3}\right)^2 (7-y)\, dy$$

(b) $\boxed{1}$ If the tank is 'half' full, i.e., the surface of the liquid oxygen is 3 m from the base, express the work required to empty the tank through the spout as an integral. **Do not evaluate the integral.** Do not reinvent the wheel, just make the needed change(s) to the integral above.

$$W = \rho g \pi \int_3^6 \left(\frac{y}{3}\right)^2 (y+1)\, dy$$

$$W = \rho g \pi \int_0^3 \left(\frac{6-y}{3}\right)^2 (7-y)\, dy$$