

Math 172 Quiz 5

12 October 2018

Sections: 7.7, 8.1

Name: _____ Point values in .

Show work. Little or no work may receive little or no credit.

Given: $\sin(2x) = 2 \sin x \cos x$ || $\sin^2 x = (1 - \cos(2x))/2$ || $\cos^2 x = (1 + \cos(2x))/2$ || $\int \sec x = \ln |\sec x + \tan x| + C$

1. [2] Fill in the blanks. For $a > 0$,

(a) the p -integral $\int_a^\infty \frac{dx}{x^p}$ converges for $P > 1$ and diverges for $P \leq 1$, and

(b) the p -integral $\int_0^a \frac{dx}{x^p}$ converges for $P < 1$ and diverges for $P \geq 1$.

2. [2] Use the Comparison Theorem to show $\int_1^\infty \frac{dx}{x^4 + 3}$ converges.

Since $0 < \frac{1}{x^4 + 3} < \frac{1}{x^4}$ & $\int_1^\infty \frac{dx}{x^4}$ is a convergent

p -integral ($p = 4 > 1$), then by comparison

$\int_1^\infty \frac{dx}{x^4 + 3}$ also converges.

3. [3] Using appropriate limit notation, evaluate the convergent integral

$$\int_0^\infty \frac{2x}{(x^2 + 3)^2} dx.$$

$$\int_0^\infty \frac{2x}{(x^2 + 3)^2} dx = \int_3^\infty \frac{du}{u^2} = \lim_{R \rightarrow \infty} \left[-\frac{1}{u} \right] \Bigg|_3^R = \lim_{R \rightarrow \infty} \left[\frac{1}{3} - \frac{1}{R} \right] = \frac{1}{3}$$

$$u = x^2 + 3$$

$$du = 2x dx$$

$$x = 0 \longleftrightarrow u = 3$$

$$x \rightarrow \infty \longleftrightarrow u \rightarrow \infty$$

4. [3] Find the area of the surface generated by rotating the graph of $y = x^3$ about the x -axis.

for $x \in [0, 1]$.

$$y = x^3$$

$$y' = 3x^2$$

$$1 + (y')^2 = 1 + 9x^4$$

$$V = 2\pi \int_0^1 x^3 \sqrt{1 + 9x^4} dx$$

$$\begin{aligned} u &= 1 + 9x^4 & 0 &\mapsto 1 \\ du &= 36x^3 dx & 1 &\mapsto 10 \end{aligned}$$

$$= \frac{\pi}{18} \int_1^{10} u^{1/2} du$$

$$= \frac{\pi}{18} \cdot \frac{2}{3} u^{3/2} \Big|_1^{10}$$

$$= \frac{\pi}{27} \left(10^{3/2} - 1 \right)$$