Math 172 Quiz 6
Sections: 10.1-10.3
25 October 2018

Name: ____________________

Point values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Please circle **True** or **False**, as appropriate.
   (a) **T / F:** The Harmonic Series converges.
   (b) **T / F:** If $a_n \to 0$ as $n \to \infty$, the series $\sum a_n$ converges.
   (c) **T / F:** $\sum n^p$ converges for $p < -1$.
   (d) **T / F:** If $\sum a_n$ converges, $a_n \to 0$ as $n \to \infty$.

2. $\boxed{1}$ For each of the following series, determine if the Test for Divergence can be used to show the series **C**onverges, **D**iverges, or is **I**nconclusive.
   (a) **C / D / I :** $\sum \frac{1}{n}$
   (b) **C / D / I :** $\sum \frac{1}{n^2}$
   (c) **C / D / I :** $\sum \frac{1}{1+(\frac{2}{3})^n}$
   (d) **C / D / I :** $\sum \sin n$

3. $\boxed{1}$ For $c \neq 0$, the Geometric Series $\sum_{n=0}^{\infty} cr^n$ converges to $\frac{c}{1-r}$ for $|r| < 1$ and diverges for $|r| \geq 1$.

4. $\boxed{2}$ For what values of x does the following converge? For those x, find the sum.

$$\sum_{n=0}^{\infty} 3(2x)^n$$

$$= \frac{3}{1-2x} \quad \text{for} \quad |2x| < 1$$

i.e., $|x| < \frac{1}{2}$

Continued on reverse.

5. $\boxed{4}$ Use the Integral Test to show the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{1+n^2}$$

Let $f(x) = \dfrac{1}{1+x^2}$.

For $x \geq 1$, $f(x)$ is positive, continuous, and decreasing

since $f'(x) = \dfrac{-2x}{(1+x^2)^2}$.

Furthermore, ~~For~~

$$\int_1^{\infty} \frac{dx}{1+x^2} = \lim_{R \to \infty} \arctan R - \arctan 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$$

By the Integral Test, since $\displaystyle\int_1^{\infty} \frac{dx}{1+x^2}$ converges

so does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+n^2}$.