Practice Final C

* NOTE: start by making the function proper.

1\. A. $$\frac{x^2-x-1}{x^2-x-6} = \frac{x^2-x-6}{x^2-x-6} + \frac{5}{x^2-x-6} = 1 + \frac{5}{x^2-x-6}$$

$$\frac{5}{x^2-x-6} = \frac{A}{x-3} + \frac{B}{x+2} \Rightarrow 5 = A(x+2) + B(x-3)$$

Let $x=-2 \Rightarrow B=-1$

Let $x=3 \Rightarrow A=1$

so

$$\int \frac{x^2-x-1}{x^2-x-6}\,dx = \int\left(1 + \frac{1}{x-3} - \frac{1}{x+2}\right)dx = x + \ln|x-3| - \ln|x+2| + C$$

B. $$\int 2x \arctan x\,dx = x^2\arctan x - \int \frac{x^2}{1+x^2}\,dx = x^2\arctan x + \int\frac{1}{1+x^2}\,dx - \int dx$$

$x^2 = (1+x^2)-1$

$u = \arctan x$ $\quad dv = 2x\,dx$

$du = \dfrac{dx}{1+x^2}$ $\quad v = x^2$

$$= x^2\arctan x + \arctan x - x + C$$

C. $$\int \tan^3\theta\sec\theta\,d\theta = \int \tan^2\theta\,(\tan\theta\sec\theta)\,d\theta = \int (u^2-1)\,du = \frac{u^3}{3} - u + C$$

$$= \frac{\sec^3\theta}{3} - \sec\theta + C$$

Let $u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

$\tan^2\theta = \sec^2\theta - 1$

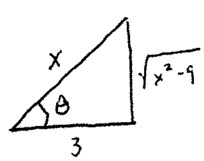
D. $$\int \frac{\sqrt{x^2-9}}{x}\,dx = \int \frac{3\tan\theta}{3\sec\theta}\cdot 3\sec\theta\tan\theta\,d\theta = \int 3\tan^2\theta\,d\theta = 3\int(\sec^2\theta-1)\,d\theta$$

Let $x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

so $x^2-9 = 3^2(\sec^2\theta-1)$

$= 3^2\tan^2\theta$



$$= 3\Big[\tan\theta - \theta\Big] + C$$

$$= 3\left(\frac{\sqrt{x^2-9}}{3} - \operatorname{arcsec}\left(\frac{x}{3}\right)\right) + C$$

$$= \cancel{3}\sqrt{x^2-9} - 3\operatorname{arcsec}\left(\frac{x}{3}\right) + C$$

NOTE: WOLFRAM ALPHA gives

$$\sqrt{x^2-9} + 3\arctan\left(\frac{3}{\sqrt{x^2-9}}\right) + \text{Const}$$

Are they both correct?

Practice Final C

2. $x = t+1$ so $t = x-1$

   $y = t^2 - 3$ so $y = (x-1)^2 - 3$

3. A. $(4 + 3\cos t,\ 5 + 3\sin t)$ for $0 \le t \le 2\pi$

   B. $(4 + 3t,\ 8 - 10t)$ for $0 \le t \le 1$

4. $x = 3t^2 + 1$, $x' = 6t$

   $y = 4t^2 - 2$, $y' = 8t$

   $\sqrt{(x')^2 + (y')^2} = \sqrt{36t^2 + 64t^2} = 10t$

   A. $s = \int_0^2 10t\,dt = 5t^2\Big|_0^2 = 20$

   B. Speed $= 10t\Big|_{t=\pi} = 10\pi$

   C. $\frac{dy}{dx} = \frac{8t}{6t} = 4/3$ for all t

5. $r = f(\theta)$ so $x = f(\theta)\cos\theta$, $y = f(\theta)\sin\theta$

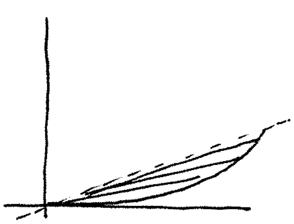
   A. $x' = f'(\theta)\cos\theta - f(\theta)\sin\theta$

   $y' = f'(\theta)\sin\theta + f(\theta)\cos\theta$

   since $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ the result follows

   B. $r = \cos 2\theta$ so $\frac{dy}{dx} = \frac{-2\sin 2\theta \sin\theta + \cos 2\theta\cos\theta}{-2\sin 2\theta\cos\theta - \cos 2\theta \sin\theta}$ so $\frac{dy}{dx}\Big|_{\theta = \frac{\pi}{6}} = \frac{\sqrt{3}}{7}$

6. By symmetry we only compute the area of



$$16 \cdot \frac{1}{2}\int_0^{\pi/8} \sin^2 2\theta\,d\theta = 4\int_0^{\pi/8} (1 - \cos 4\theta)\,d\theta$$

$$= 4\left(\theta - \frac{1}{4}\sin 4\theta\right)\Big|_0^{\pi/8} = \frac{\pi}{2} - 1$$

//