Practice Final D

1. A. $\displaystyle\int \frac{(x+1)\ln x}{x^2}\,dx = \int \frac{\ln x}{x}\,dx + \int \frac{\ln x}{x^2}\,dx = \frac{1}{2}(\ln x)^2 - \frac{\ln x}{x} + \int x^{-2}\,dx$

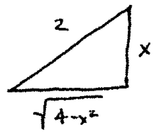
$u = \ln x$, $du = \frac{1}{x}dx$ $\Big\}$ $u = \ln x$, $dv = x^{-2}dx$, $du = \frac{1}{x}dx$, $v = -x^{-1}$

$\displaystyle = \frac{1}{2}(\ln x)^2 - \frac{\ln x}{x} - \frac{1}{x} + C$

B. $\displaystyle\int \frac{\sqrt{4-x^2}}{x}\,dx = \int \frac{2\cos\theta}{2\sin\theta}\,2\cos\theta\,d\theta = 2\int \frac{\cos^2\theta}{\sin\theta}\,d\theta = 2\int \frac{(1-\sin^2\theta)}{\sin\theta}\,d\theta = 2\int(\csc\theta - \sin\theta)\,d\theta$

$x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$



$= 2\left[\cos\theta - \ln|\csc\theta + \cot\theta|\right] + C$

$\displaystyle = 2\left[\frac{\sqrt{4-x^2}}{2} - \ln\left|\frac{2}{x} + \frac{\sqrt{4-x^2}}{x}\right|\right] + C$

C. $\displaystyle\int \sin^3\theta\cos\theta\,d\theta = \frac{1}{4}\sin^4\theta + C$

D. $\displaystyle\int \frac{5x+7}{(x+1)(x+2)}\,dx = \int\left(\frac{2}{x+1} + \frac{3}{x+2}\right)dx = 2\ln|x+1| + 3\ln|x+2| + C$

$\displaystyle\frac{5x+7}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$5x+7 = A(x+2) + B(x+1)$

let $x = -1 \Rightarrow A = 2$

$x = -2 \Rightarrow B = 3$

2. $x = 1 + \cos t \Rightarrow \cos t = x - 1$

$y = \cos^3 t \Rightarrow y = (x-1)^3$

3. $x = e^{-t}\cos t$ $\qquad y = e^{-t}\sin t$

$x' = -e^{-t}\cos t - e^{-t}\sin t$ $\qquad y' = -e^{-t}\sin t + e^{-t}\cos t$

$(x')^2 + (y')^2 = e^{-2t}\left[\cos^2 t + 2\cos t\sin t + \sin^2 t + \sin^2 t - 2\cos t\sin t + \cos^2 t\right] = 2e^{-2t}$

A) $\displaystyle s = \int_0^1 \sqrt{2}e^{-t} = -\sqrt{2}e^{-t}\Big|_0^1 = \sqrt{2}\left(1 - \frac{1}{e}\right)$

B) $\displaystyle \sqrt{2e^{-2t}}\Big|_{t=0} = \sqrt{2}$

C) $\displaystyle \frac{dy}{dx}\Big|_{t=0} = \frac{1}{-1} = -1$

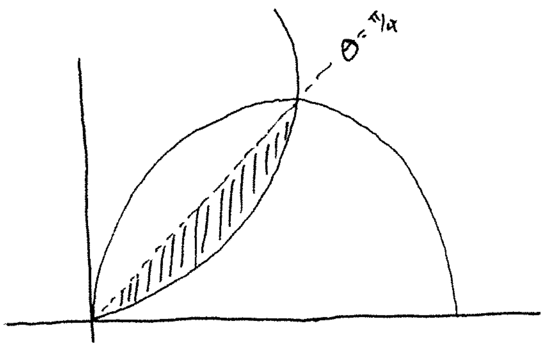
4. $r = \dfrac{2}{3\sin\theta + 4\cos\theta} \Rightarrow 3r\sin\theta + 4r\cos\theta = 2 \Rightarrow 3y + 4x = 2$

5. $r = e^{\theta}$

$r' = e^{\theta}$ $\qquad r^2 + (r')^2 = 2e^{2\theta}$

$$S = \int_0^{2\pi} \sqrt{2}\, e^{\theta}\, d\theta = \sqrt{2}\, e^{\theta}\Big|_0^{2\pi} = \sqrt{2}\left(e^{2\pi} - 1\right)$$

6.



By symmetry, we find the shaded area & multiply by two.

$$\text{Area} = 2 \cdot \frac{1}{2} \int_0^{\pi/4} \sin^2\theta \, d\theta = \frac{1}{2}\int_0^{\pi/4} (1 - \cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]\Bigg|_0^{\pi/4} = \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right)$$