Math 172 Exam 1 Review Problem Solutions

1. A. $\displaystyle\int_0^3 t^2\sqrt{1+t}\,dt = \int_1^4 (u-1)^2\sqrt{u}\,du = \int_1^4 \left(u^{5/2} - 2u^{3/2} + u^{1/2}\right)du = \frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2}\Big|_1^4$

let $u = 1+t$, $du = dt$

t	$\longmapsto$	u
0		1
3		4

$= \frac{2}{7}(128-1) - \frac{4}{5}(32-1) + \frac{2}{3}(8-1)$ ← stop here

$= \frac{1696}{105}$

B. $\displaystyle\int \frac{dy}{y^2+4} = \frac{1}{4}\int \frac{dy}{\left(\frac{y}{2}\right)^2+1} = \frac{1}{2}\int \frac{du}{u^2+1} = \frac{1}{2}\arctan\left(\frac{y}{2}\right) + C$

let $u = y/2$
$du = \frac{1}{2}\,dy$

C. $\displaystyle\int x\arctan x\,dx = \frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx = \frac{x^2}{2}\arctan x - \frac{1}{2}\int\left(\underbrace{\frac{x^2+1}{x^2+1}}_{=1} - \frac{1}{x^2+1}\right)dx$

$u = \arctan x$ $\quad dv = x\,dx$

$du = \frac{1}{1+x^2}\,dx$ $\quad v = \frac{x^2}{2}$

$= \frac{x^2}{2}\arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + C$

D. $\displaystyle\int_0^{\pi/2} \sin^3 2\theta\,d\theta = \int_0^{\pi/2}\left(1-\cos^2 2\theta\right)\sin 2\theta\,d\theta = -\frac{1}{2}\int_1^{-1}(1-u^2)\,du = -\frac{1}{2}\left(u - \frac{u^3}{3}\right)\Big|_1^{-1}$

$u = \cos 2\theta$
$du = -2\sin 2\theta\,d\theta$

θ	$\longmapsto$	u
0		1
$\pi/2$		-1

$= -\frac{1}{2}\left(-1 + \frac{1}{3} - 1 + \frac{1}{3}\right)$

$= \frac{2}{3}$

2. A $\displaystyle\int_1^{e^\pi} \frac{\sin(\ln t)}{t}\,dt = \int_0^\pi \sin u\,du = -\cos u\Big|_0^\pi = 2$

let $u = \ln t$
$du = \frac{1}{t}\,dt$

t	$\longmapsto$	u
1		0
e^π		π

B. $\displaystyle\int \frac{dy}{\sqrt{2-y^2}} = \frac{1}{\sqrt{2}}\int \frac{dy}{\sqrt{1-(y/\sqrt{2})^2}} = \int \frac{du}{\sqrt{1-u^2}} = \arcsin\left(\frac{y}{\sqrt{2}}\right) + C$

let $u = y/\sqrt{2}$ so $du = \frac{1}{\sqrt{2}}\,dy$

2.c. $\displaystyle\int x \ln x \, dx = \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^2}{2}$

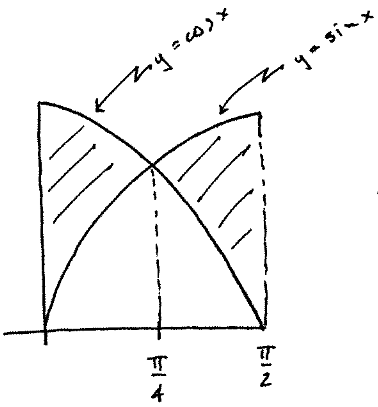
D. $\displaystyle\int_0^{\pi/4} \sec^4\theta \, d\theta = \int_0^{\pi/4} (1+\tan^2\theta)\sec^2\theta \, d\theta = \int_0^1 (1+u^2)\,du = u + \frac{u^3}{3}\Big|_0^1 = \frac{4}{3}$

let $u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$\theta \mapsto$	u
0	0
$\pi/4$	1

3.

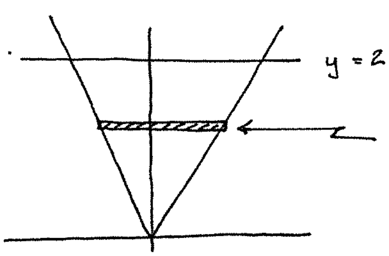


$\displaystyle \text{Area} = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$

$\displaystyle = 2\int_0^{\pi/4} (\cos x - \sin x)\,dx$ by symmetry

$\displaystyle = 2(\sin x + \cos x)\Big|_0^{\pi/4} = 2\left[\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - 1\right] = 2\left[\sqrt{2} - 1\right]$

4.



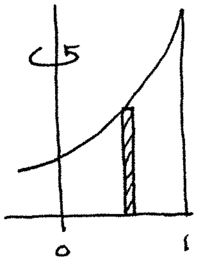
$V_i = (\text{side})^2 \Delta y$

$= (2x)^2 \Delta y$

$= y^2 \Delta y$

so $\displaystyle V = \int_0^2 y^2\,dy = \frac{y^3}{3}\Big|_0^2 = \frac{8}{3}$

5.



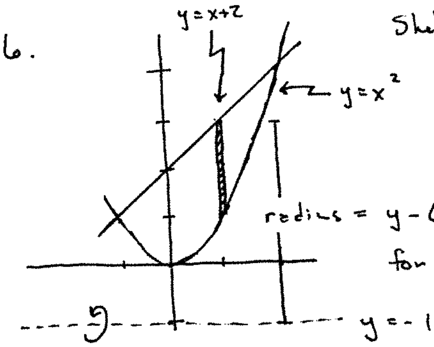
Using washers would require two integrals & would involve $(1)^2 - (\ln y)^2$ which is ugly! We should use shells.

$\displaystyle 2\pi \int_0^1 x e^x \, dx = 2\pi\left(x e^x\Big|_0^1 - \int_0^1 e^x\,dx\right) = 2\pi\left(e - e^x\Big|_0^1\right) = 2\pi\,(e - e + 1) = 2\pi$

$u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

6.



Shells would require two integrals, let's use washers.

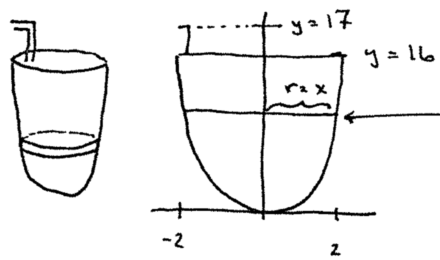
$$V = \pi \int_{-1}^{2} \left[(x+2+1)^2 - (x^2+1)^2 \right] dx = \pi \int_{-1}^{2} (8 + 6x - x^2 - x^4)\, dx$$

$$= \pi \left(8x + 3x^2 - \frac{x^3}{3} - \frac{x^5}{5} \right) \Bigg|_{-1}^{2}$$

$$= \pi \left[\left(16 + 12 - \frac{8}{3} - \frac{32}{5} \right) - \left(-8 + 3 + \frac{1}{3} + \frac{1}{5} \right) \right] \quad \longleftarrow \text{stop here}$$

$$= \frac{117\pi}{5}$$

7.



$$W_i = \rho g \pi\, (\text{radius})^2 \overbrace{(17-y)}^{\text{distance}} \Delta y$$

$$= \rho g \pi\, x^2 (17-y) \Delta y \qquad \text{but } y = x^4 \text{ so } \sqrt{y} = x^2$$

$$= \rho g \pi \sqrt{y}\, (17-y) \Delta y$$

$$\text{Work} = \rho g \pi \int_0^{16} \left(17\sqrt{y} - y^{3/2} \right) dy = \rho g \pi \left(\frac{34}{3} y^{3/2} - \frac{2}{5} y^{5/2} \right) \Bigg|_0^{16}$$

$$= \rho g \pi \left(\frac{34}{3} 16^{3/2} - \frac{2}{5} y^{5/2} \right)$$

8\. A. $\displaystyle\int \frac{t\,dt}{\sqrt{4-t^2}} = -\frac{1}{2}\int u^{-1/2}\,du = -u^{1/2} + C = C - \sqrt{4-t^2}$

let $u = 4-t^2$

$du = -2t\,dt$

B. $\displaystyle\int_{1/8}^{1/4} \frac{dy}{\sqrt{1-16y^2}} = \frac{1}{4}\int_{1/2}^{1} \frac{du}{\sqrt{1-u^2}} = \frac{1}{4}\arcsin u \Big|_{1/2}^{1} = \frac{1}{4}\left(\frac{\pi}{2}-\frac{\pi}{6}\right) = \frac{\pi}{12}$

let $u = 4y$ $\quad \begin{array}{cc} y \longmapsto & u \\ \hline 1/8 & 1/2 \\ 1/4 & 1 \end{array}$

$du = 4\,dy$

C. $\displaystyle\int_0^{\pi/4} x\sec^2 x\,dx = x\tan x\Big|_0^{\pi/4} - \int_0^{\pi/4} \tan x\,dx = \frac{\pi}{4} - \ln|\sec x|\Big|_0^{\pi/4} = \frac{\pi}{4} - \ln\sqrt{2}$

$u = x \qquad dv = \sec^2 x\,dx$

$du = dx \qquad v = \tan x$

D. $\displaystyle\int \sin^4\theta\,d\theta = \int\left[\tfrac{1}{2}(1-\cos 2\theta)\right]^2 d\theta = \frac{1}{4}\int\left(1 - 2\cos 2\theta + \cos^2 2\theta\right)d\theta$

$\displaystyle = \frac{1}{4}\int\left(1 - 2\cos 2\theta + \frac{1}{2} + \frac{1}{2}\cos 4\theta\right)d\theta = \frac{1}{4}\left(\frac{3\theta}{2} - \sin 2\theta + \frac{1}{8}\sin 4\theta\right) + C$

9\. A. $\displaystyle\int t\sqrt{2-t}\,dt = -\int (2-u)\sqrt{u}\,du = \int\left(u^{3/2} - 2u^{1/2}\right)du = \frac{2}{5}(2-t)^{5/2} - \frac{4}{3}(2-t)^{3/2} + C$

let $u = 2-t$

$du = -dt$

B. $\displaystyle\int_0^{1/3} \frac{dy}{9y^2+1} = \frac{1}{3}\int_0^1 \frac{du}{u^2+1} = \frac{1}{3}\arctan u\Big|_0^1 = \frac{1}{3}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{12}$

Let $3y = u$ $\quad \begin{array}{cc} y \longmapsto & u \\ \hline 0 & 0 \\ 1/3 & 1 \end{array}$

$3\,dy = du$

9.c. $\int x^2 \sin 2x\, dx = -\frac{x^2}{2}\cos 2x + \int x\cos 2x\, dx = -\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x - \int \frac{1}{2}\sin 2x\, dx$

$u = x^2 \quad dv = \sin 2x\, dx$

$du = 2x\,dx \quad v = -\frac{1}{2}\cos 2x$

$\left\{ \begin{array}{ll} u = x & dv = \cos 2x\, dx \\ du = dx & v = \frac{1}{2}\sin 2x \end{array} \right.$

$= -\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos x + C$

D. $\int \sec^4\theta \tan\theta\, d\theta = \int \sec^3\theta \sec\theta\tan\theta\, d\theta = \int u^3\, du = \frac{1}{4}\sec^4\theta + C$

$u = \sec\theta$

$du = \sec\theta\tan\theta\, d\theta$

—— OR ——

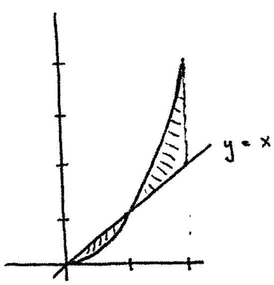
$= \int (1+\tan^2\theta)\tan\theta\sec^2\theta\, d\theta = \int (1+u^2)u\, du = \frac{u^2}{2} + \frac{u^4}{4} + C$

$u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$= \frac{1}{2}\tan^2\theta + \frac{1}{4}\tan^4\theta + C$

10.

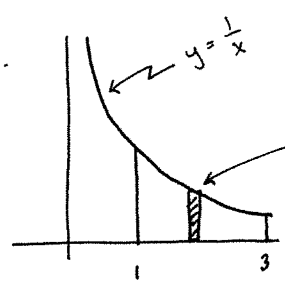


$A = \int_0^1 (x - x^2)\, dx + \int_1^2 (x^2 - x)\, dx = \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 + \left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_1^2$

$= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - \frac{1}{2}\right)$ ← stop

$= 1$

11.

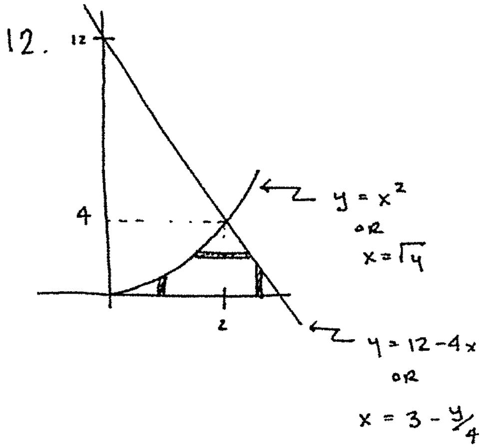


$V_i = \frac{\pi}{2}(\text{radius})^2 \Delta x$ where the radius $= \frac{y}{2} = \frac{1}{2x}$

$= \frac{\pi}{8} x^{-2} \Delta x$

$\text{Volume} = \frac{\pi}{8}\int_1^3 x^{-2}\, dx = -\frac{\pi}{8}x^{-1}\Big|_1^3 = \frac{\pi}{8}\left(1 - \frac{1}{3}\right) = \frac{\pi}{12}$

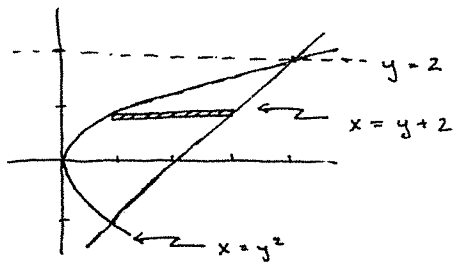
12.



$$V_{Disk} = \pi \int_0^2 (x^2)^2 \, dx + \pi \int_2^3 (12 - 4x)^2 \, dx$$

$$V_{Shell} = 2\pi \int_0^4 (y)\left(3 - \frac{y}{4} - \sqrt{y}\right) dy$$

13.



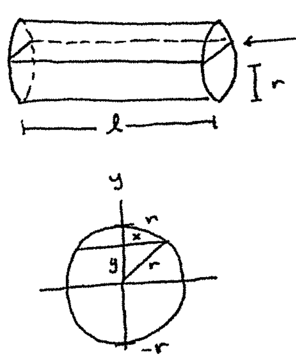
Washers would require two integrals, so we use shells.

$$V = 2\pi \int_{-1}^{2} (2 - y)(y + 2 - y^2)\, dy = 2\pi \int_{-1}^{2} (4 - 3y^2 + y^3)\, dy$$

$$= 2\pi \left(4y - y^3 + \frac{y^4}{4}\right)\Big|_{-1}^{2} = 2\pi \left(8 - 8 + 4 - \left(-4 + 1 + \frac{1}{4}\right)\right)$$

$$= 2\pi \left(\frac{27}{4}\right) = \frac{27\pi}{2}$$

14.



Work for this slice

$= \rho g \, (\text{width})(\text{length})(\text{distance}) \, \Delta y$

$= \rho g \, (2x)(l)(r - y) \, \Delta y$

$= \rho g \, (2\sqrt{r^2 - y^2}) \, l \, (r - y) \, \Delta y$

$= 2 \rho g \, l \, (r - y) \sqrt{r^2 - y^2} \, \Delta y$

$x^2 + y^2 = r^2$

so $x = \sqrt{r^2 - y^2}$

odd functions so $= 0$

$$W = 2\rho g l \int_{-r}^{r} (r - y)\sqrt{r^2 - y^2}\, dy = 2\rho g l \left[\int_{-r}^{r} r\sqrt{r^2 - y^2}\, dy - \int_{-r}^{r} y\sqrt{r^2 - y^2}\, dy\right]$$

$$= 2\rho g l r \int_{-r}^{r} \sqrt{r^2 - y^2}\, dy$$

Area of semicircle

$$= 2\rho g l r \left(\frac{\pi r^2}{2}\right) = \pi \rho g l r^3$$

Additional Problems

1. A. $\displaystyle\int_1^e \frac{\ln x}{x}\,dx = \int_0^1 u\,du = \frac{1}{2}u^2\Big|_0^1 = \frac{1}{2}$

$u = \ln x$ $\quad$ $du = \frac{1}{x}\,dx$

x	$\mapsto$	u
1		0
e		1

Note: We could also use parts,

$$\int_1^e \frac{\ln x}{x}\,dx = (\ln x)^2\Big|_1^e - \int_1^e \frac{\ln x}{x}\,dx \quad \text{so} \quad 2\int_1^e \frac{\ln x}{x}\,dx = 1 \quad \text{or} \quad \int_1^e \frac{\ln x}{x}\,dx = \frac{1}{2}$$

$u = \ln x$ $\quad$ $dv = \frac{1}{x}\,dx$

$du = \frac{1}{x}\,dx$ $\quad$ $v = \ln x$

B. $$\int_0^3 \frac{1+2x}{9+x^2}\,dx = \int_0^3 \frac{dx}{9+x^2} + \int_0^3 \frac{2x}{9+x^2}\,dx = \frac{1}{9}\int_0^3 \frac{dx}{1+(x/3)^2} + \int_9^{18} \frac{du}{u}$$

use a inverse trig function with

$t = \frac{x}{3}$ $\quad$ $0 \mapsto 0$

$dt = \frac{1}{3}\,dx$ $\quad$ $3 \mapsto 1$

let $u = 9 + x^2$ $\quad$ $0 \mapsto 9$

$du = 2x\,dx$ $\quad$ $3 \mapsto 18$

$$= \frac{1}{3}\int_0^1 \frac{dt}{1+t^2} + \ln u\Big|_9^{18}$$

$$= \frac{1}{3}\arctan t\Big|_0^1 + \ln 18 - \ln 9$$

$$= \frac{\pi}{12} + \ln 2$$

C. $$\int_0^1 \frac{1+x}{\sqrt{1-x^2}}\,dx = \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx + \frac{1}{2}\int_0^1 \frac{2x}{\sqrt{1-x^2}}\,dx \longleftarrow$$

$u = 1 - x^2$ $\quad$ $du = -2x\,dx$

x	$\longmapsto$	u
0		1
1		0

$$= \arcsin x\Big|_0^1 + [redacted]\sqrt{u}\Big|_0^1 = \frac{\pi}{2} + [redacted]1$$

1\. D. $\displaystyle\int_0^{\pi/2} x\cos x\,dx = x\sin x\Big|_0^{\pi/2} - \int_0^{\pi/2}\sin x\,dx = \frac{\pi}{2} + \cos x\Big|_0^{\pi/2} = \frac{\pi}{2} - 1$

$u = x \qquad dv = \cos x\,dx$

$du = dx \qquad v = \sin x$

E. $\displaystyle\int e^{2x}\sin 3x\,dx = \frac{1}{2}e^{2x}\sin 3x - \int \frac{3}{2}e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\sin 3x - \frac{3}{4}e^{2x}\cos 3x - \int \frac{9}{4}e^{2x}\sin 3x\,dx$

$u = \sin 3x \qquad dv = e^{2x}\,dx$

$du = 3\cos 3x\,dx \qquad v = \frac{1}{2}e^{2x}$

$\left\{\begin{array}{ll} u = \cos 3x & dv = \frac{3}{2}e^{2x}\,dx \\ du = -3\sin 3x\,dx & v = \frac{3}{4}e^{2x} \end{array}\right.$

so $\displaystyle\frac{13}{4}\int e^{2x}\sin 3x\,dx = \frac{1}{4}e^{2x}\left(2\sin 3x - 3\cos 3x\right) + C$

so $\displaystyle\int e^{2x}\sin 3x\,dx = \frac{1}{13}e^{2x}\left(2\sin 3x - 3\cos 3x\right) + C$

F. $\displaystyle\int \sin x\cos x\,\sin 2x\,dx = \frac{1}{2}\int \sin^2 2x\,dx = \frac{1}{4}\int\left(1 - \cos 4x\right)dx = \frac{1}{4}\left(x - \frac{1}{4}\sin 4x\right) + C$

$\sin 2x = 2\sin x\cos x$

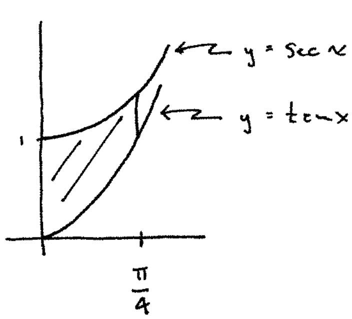
G. $\displaystyle\int \sec^3 x\tan^3 x\,dx = \int \sec^2 x\left(\sec^2 x - 1\right)\sec x\tan x\,dx = \int u^2(u^2-1)\,du$

$u = \sec x$

$du = \sec x\tan x\,dx$

$\displaystyle = \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

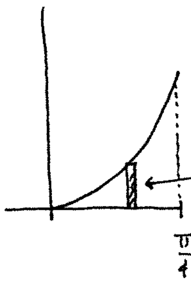
2\.



$\displaystyle \text{Area} = \int_0^{\pi/4}\left(\sec x - \tan x\right)dx = \ln|\sec x + \tan x| - \ln|\sec x|\Big|_0^{\pi/4}$

$= \ln|\sqrt{2} + 1| - \ln|\sqrt{2}|$

3.

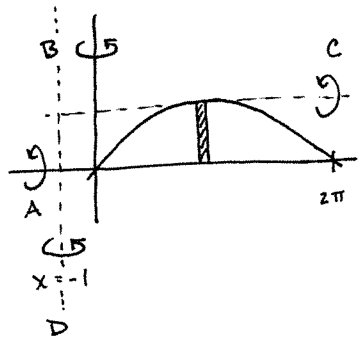


$V_i = (\text{side})^2 \Delta x$
$= \tan^2 x \, \Delta x$

$$\text{Volume} = \int_0^{\pi/4} \tan^2 x \, dx = \int_0^{\pi/4} (\sec^2 x - 1)\, dx$$

$$= \tan x - x \Big|_0^{\pi/4} = 1 - \frac{\pi}{4} = 1 - \pi/4$$

4.



$$A.\quad \pi \int_0^{2\pi} \sin^2\left(\frac{x}{2}\right) dx = \frac{\pi}{2} \int_0^{2\pi} \left((1 - \cos x)\, dx \right) = \frac{\pi}{2}\left(x - \sin x\right)\Big|_0^{2\pi} = \pi^2$$

$$B.\quad 2\pi \int_0^{2\pi} x \sin\left(\frac{x}{2}\right) dx = 2\pi\left(-2x\cos\frac{x}{2}\Big|_0^{2\pi} + 2\int_0^{2\pi} \cos\frac{x}{2}\, dx\right)$$

$u = x \qquad dv = \sin\frac{x}{2}\, dx$

$du = dx \qquad v = -2\cos\frac{x}{2}$

$$= 2\pi\left(4\pi + \sin\frac{x}{2}\Big|_0^{2\pi}\right) = 8\pi^2$$

$$C.\quad \pi \int_0^{2\pi} \left(1 - \left(1 - \sin\left(\frac{x}{2}\right)\right)^2\right) = \pi \int_0^{2\pi} \left[1 - 1 + 2\sin\frac{x}{2} - \sin^2\frac{x}{2}\right] dx$$

$$= \pi \int_0^{2\pi} \left(2\sin\frac{x}{2} - \frac{1}{2} + \cos x\right) dx = \pi\left(-4\cos\frac{x}{2} - \frac{1}{2}x + \sin x\right)\Big|_0^{2\pi}$$

$$= \pi\left(8 - \pi\right)$$

$$D.\quad 2\pi \int_0^{2\pi} (x+1)\sin\frac{x}{2}\, dx = 2\pi\left((x+1)\left(-2\cos\frac{x}{2}\right)\Big|_0^{2\pi} + 2\int_0^{2\pi} \cos\frac{x}{2}\, dx\right)$$

$u = x + 1 \qquad dv = \sin\frac{x}{2}\, dx$

$du = dx \qquad v = -2\cos\frac{x}{2}$

$$= 2\pi\left((2\pi + 1)(2) - (1)(-2) + \sin\frac{x}{2}\Big|_0^{2\pi}\right) = 8\pi(\pi + 1)$$

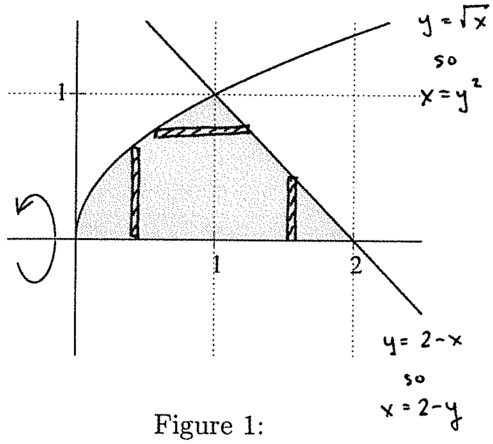


Figure 1:

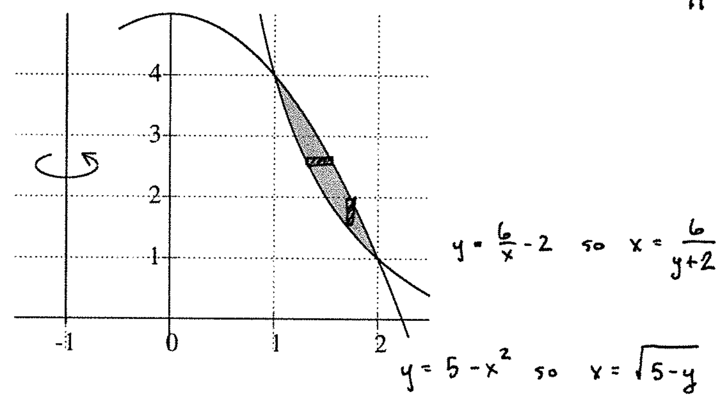


Figure 2:

5. $$V_{Disk} = \pi \int_0^1 (\sqrt{x})^2\, dx + \pi \int_1^2 (2-x)^2\, dx$$

$$V_{shell} = 2\pi \int_0^1 y\left(2 - y - y^2\right) dy = 2\pi \int_0^1 \left(2y - y^2 - y^3\right) dy = 2\pi \left(y^2 - \frac{y^3}{3} - \frac{y^4}{4} \right)\Bigg|_0^1$$

$$= 2\pi \left(1 - \frac{1}{3} - \frac{1}{4} \right) = \frac{5\pi}{6}$$

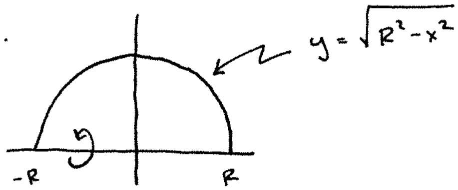
6. $$V_{Disk} = \pi \int_1^4 \left[\left(\sqrt{5-y} + 1 \right)^2 - \left(\frac{6}{y+2} + 1 \right)^2 \right] dy$$

$$V_{shell} = 2\pi \int_1^2 (x+1)\left(5 - x^2 - \frac{6}{x} + 2 \right) dx = 2\pi \int_1^2 \left(1 + 7x - x^2 - x^3 - \frac{6}{x} \right) dx$$

$$= 2\pi \left(x + \frac{7}{2}x^2 - \frac{x^3}{3} - \frac{x^4}{4} - 6 \ln |x| \right)\Bigg|_1^2 = 2\pi \left[2 + 14 - \frac{8}{3} - 4 - 6 \ln 2 - 1 - \frac{7}{2} + \frac{1}{3} + \frac{1}{4} \right]$$

$$= 2\pi \left(\frac{65}{12} - 6 \ln 2 \right)$$

7.

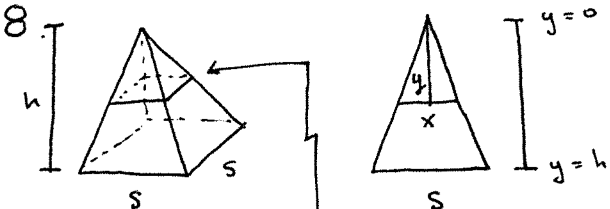


We use the Disk method to find the volume of the sphere formed by rotating the semicircle $y=\sqrt{R^2-x^2}$ about the x-axis.

$$\text{Volume} = \pi\int_{-R}^{R}\left(\sqrt{R^2-x^2}\right)^2 dx = \underbrace{2\pi\int_{0}^{R}}_{\text{Symmetry}}(R^2-x^2)\,dx = 2\pi\left(R^2x-\frac{x^3}{3}\right)\Bigg|_0^R = 2\pi\left(R^3-\frac{R^3}{3}\right)$$

$$= \frac{4}{3}\pi R^3$$

8.



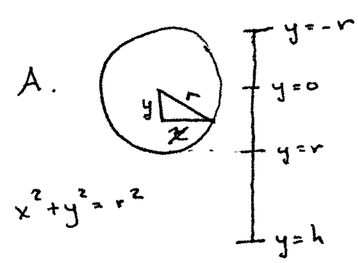
$$\frac{x}{y} = \frac{s}{h} \qquad \text{so} \qquad x = \frac{s}{h}y$$

$$W_i = \rho g\,(\text{side})^2\,\Delta y\,(\text{distance}) = \rho g\left(\frac{s}{h}y\right)^2(h-y)\,\Delta y$$

$$\text{Work} = \rho g\,\frac{s^2}{h^2}\int_0^h (hy^2-y^3)\,dy = \rho g\,\frac{s^2}{h^2}\left(\frac{hy^3}{3}-\frac{y^4}{4}\right)\Bigg|_0^h = \rho g\,\frac{s^2}{h^2}\left(\frac{h^4}{3}-\frac{h^4}{4}\right)$$

$$= \rho g\,\frac{s^2h^2}{12}$$

9. A.



$$W_i = \rho g\,\pi x^2\,\Delta y\,(h-y) = \rho g\pi\,(r^2-y^2)(h-y)\,\Delta y$$

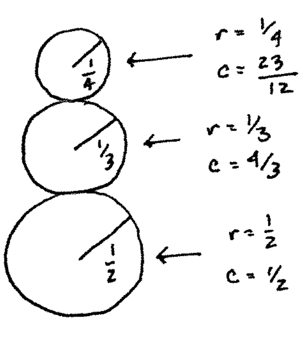
$$\text{Work} = \rho g\pi\int_{-r}^{r}(r^2h-r^2y-hy^2+y^3)\,dy$$

$$= \rho g\pi\left[r^2hy-\frac{r^2}{2}y^2-\frac{h}{3}y^3+\frac{y^4}{4}\right]_{-r}^{r}$$

$$= \rho g\pi\left[2r^3h-\frac{2h}{3}r^3\right] = \frac{4}{3}\pi r^3 h\rho g$$

Note: this is the distance the center is above the ground times the volume times ρ times g.

B.



$$W = \frac{4}{3}\pi\rho g\left[\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)^3\left(\frac{4}{3}\right)+\left(\frac{1}{4}\right)^3\left(\frac{23}{12}\right)\right]$$