Math 172 Quiz 10 Name: ____________________

1 Apr 2016 Show Appropriate Work Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Please indicate **T**rue or **F** false.
   (a) **T** / **F** : If $a_n \to 0$ as $n \to \infty$, the series $\sum a_n$ converges.
   (b) **T** / **F** : If $a_n \to 0$ as $n \to \infty$, the series $\sum a_n$ neither converges nor diverges.
   (c) **T** / **F** : If $\sum |a_n|$ converges, then $\sum a_n$ converges absolutely.
   (d) **T** / **F** : If $\sum |a_n|$ diverges, then $\sum a_n$ diverges absolutely.

2. $\boxed{4}$ Use the Limit Comparison Test to show the following series converges or diverges.

$$\sum_{n=4}^{\infty} \frac{\sqrt{n^3+7n}}{n^2+7n}$$

We consider the series $\sum_{n=4}^{\infty} \frac{n^{3/2}}{n^2} = \sum_{n=4}^{\infty} \frac{1}{\sqrt{n}}$. Since both the given series & the series we are considering are positive the L.C.T. applies.

Computing the limit gives, $\lim_{n\to\infty} \left(\frac{\sqrt{n^3+7n}}{n^2+7n} \Big/ \frac{1}{\sqrt{n}} \right) = 1$. Since the limit is positive & finite the series behave the same. We know $\sum_{n=4}^{\infty} \frac{1}{\sqrt{n}}$ is a divergent p-series, ~~s~~ so by the Limit Comparison Test,

$$\sum_{n=4}^{\infty} \frac{\sqrt{n^3+7n}}{n^2+7n} \text{ also diverges.}$$

3. $\boxed{4}$ Use the Integral Test to show the following series converges or diverges. Be sure to verify the hypotheses.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

1 $\left\{ \right.$ Let $f(x) = \frac{1}{x\sqrt{\ln x}}$

f is continuous & positive for $x > 1$.

$f' = -\left(x\sqrt{\ln x}\right)^{-2}\left(\sqrt{\ln x} + x \cdot \frac{1}{2}(\ln x)^{-1/2} \cdot \frac{1}{x}\right) < 0$

for $x > 1$, i.e. f is decreasing.

2 $\left\{ \right.$ We now consider the integral $\int_2^{\infty} \frac{dx}{x(\ln x)^{1/2}} = \int_{\ln 2}^{\infty} \frac{du}{u^{1/2}}$ which is a divergent p-integral.

Let $u = \ln x$

$du = \frac{1}{x} dx$

1 $\left\{ \right.$ By the integral test, $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ also diverges