Math 172 | Quiz | Name: ____________

2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Please indicate **T**rue or **F** false.

   (a) **T** / **F** : If $\sum a_n(x-3)^n$ converges for $x=4$, then $\sum a_n(x-3)^n$ converges for $x=1$.

   (b) **T** / **F** : If $\sum a_n(x-3)^n$ converges for $x=1$, then $\sum a_n(x-3)^n$ converges for $x=4$.

2. $\boxed{4}$ Find the **Interval of Convergence** for the following power series.

$$\sum_{n=1}^{\infty}\frac{(x-2)^n}{n2^n}$$

Find R using the Root Test: $\sqrt[n]{\left|\frac{(x-2)^n}{n\,2^n}\right|} \xrightarrow[n\to\infty]{} \frac{|x-2|}{2} < 1$ so $|x-2|<2$

Now check endpoints.

$x=0$: $\sum_{n=1}^{\infty}\frac{(-2)^n}{n\,2^n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ which converges by the A.S.T

$x=4$: $\sum_{n=1}^{\infty}\frac{2^n}{n2^n} = \sum_{n=1}^{\infty}\frac{1}{n}$ which diverges (Harmonic).

so $I = [0,4)$

3. $\boxed{4}$ Given that

$$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n \quad \text{for} \quad |x|<1,$$

find a power series representation and the interval on which it is valid for the following function.

$$f(x)=\frac{4}{4+x^2} = \frac{1}{1+\frac{x^2}{4}} = \frac{1}{1-\left(\frac{-x^2}{4}\right)}$$

so $f(x) = \sum_{n=0}^{\infty}\left(\frac{-x^2}{4}\right)^n = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{4^n}$

for $\left|\frac{-x^2}{4}\right|<1$ i.e. $|x|<2$, or $(-2,2)$