

1. Integrate.

(a) 3 $\int (2x+1) \sin 3x \, dx$ $= (2x+1) \left(-\frac{1}{3} \cos 3x\right) + \int \frac{2}{3} \cos 3x \, dx$

$u = 2x+1$ $dv = \sin 3x \, dx$

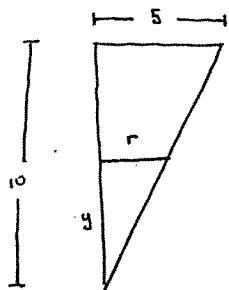
$du = 2 \, dx$ $v = -\frac{1}{3} \cos 3x$ $= (2x+1) \left(-\frac{1}{3} \cos 3x\right) + \frac{2}{9} \sin 3x + C$

(b) 3 $\int_1^e x^2 \ln x \, dx$ $= \ln x \cdot \frac{x^3}{3} \Big|_1^e - \int_1^e \frac{x^2}{3} \, dx = \frac{e^3}{3} - \frac{x^3}{9} \Big|_1^e = \frac{e^3}{3} - \frac{e^3}{9} + \frac{1}{9}$

$u = \ln x$ $dv = x^2 \, dx$

$du = \frac{1}{x} \, dx$ $v = \frac{x^3}{3}$ $= \frac{2e^3 + 1}{9}$

2. 4 Express, as an integral, the work (in joules) required to pump all of the water out of the full conical tank in the figure below; water exits through the spout. Distances are in meters, the density of water is ρ , acceleration due to gravity is g . Clearly specify the coordinate system you are using. **You do not need to evaluate the integral.**



$\frac{y}{r} = \frac{5}{10}$ so $r = \frac{1}{2}y$

$\int \rho g \pi (\text{radius})^2 \Delta y (12-y) = W;$

$\frac{1}{4} \rho g \pi y^2 (12-y) \Delta y$

$W_{\text{work}} = \frac{1}{4} \rho g \pi \int_0^{10} y^2 (12-y) \, dy$

