Math 172 | Quiz 4 | Name: ______________
19 Feb 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{3}$ When using trigonometric substitution we discussed three basic forms of substitution. For integrals involving the following forms, state the appropriate substitution, i.e. $x = blah$, and the appropriate change of variables term, i.e. $dx = stuff$.

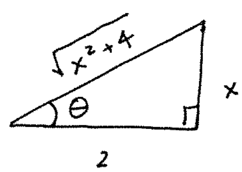
   (a) $\sqrt{a^2 + x^2}$ $\quad x = a\tan\theta \quad dx = a\sec^2\theta\, d\theta$

   (b) $\sqrt{x^2 - a^2}$ $\quad x = a\sec\theta \quad dx = a\sec\theta\tan\theta\, d\theta$

   (c) $\sqrt{a^2 - x^2}$ $\quad x = a\sin\theta \quad dx = a\cos\theta\, d\theta$

2. $\boxed{3}$ If $x = 2\tan\theta$, use an appropriate triangle to find $\sin\theta$.

$\tan\theta = \frac{x}{2}$



$\sin\theta = \dfrac{x}{\sqrt{x^2+4}}$

3. $\boxed{4}$ Integrate the following using an appropriate trigonometric substitution.

$$\int_{\sqrt{3}}^{2} \frac{\sqrt{x^2-3}}{x}\,dx = \int_0^{\pi/6} \frac{\sqrt{3}\tan\theta}{\sqrt{3}\sec\theta}\sqrt{3}\sec\theta\tan\theta\, d\theta$$

Let $x = \sqrt{3}\sec\theta$

$dx = \sqrt{3}\sec\theta\tan\theta\, d\theta$

$$= \int_0^{\pi/6} \sqrt{3}\tan^2\theta\, d\theta = \sqrt{3}\int_0^{\pi/6} (\sec^2\theta - 1)\, d\theta$$

$x = \sqrt{3} \longmapsto \theta = \operatorname{arcsec}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = 0$

$x = 2 \longmapsto \theta = \operatorname{arcsec}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$

$$= \sqrt{3}\left(\tan\theta - \theta\right)\Big|_0^{\pi/6} = \sqrt{3}\left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right)$$

so $x^2 - 3 = 3\sec^2\theta - 3$

$= 3(\sec^2\theta - 1)$

$= 3\tan^2\theta$

$$= 1 - \frac{\pi\sqrt{3}}{6}$$