Math 172 | Quiz 6 | Name: ______________
1 Mar 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{3}$ Using an appropriate limit, evaluate the following convergent integral.

$$\int_0^\infty xe^{-x^2}\,dx$$

Let $u = -x^2$, $du = -2x\,dx$

$x = 0 \longmapsto u = 0$

$x \to \infty \longmapsto u \to -\infty$

$$= \frac{1}{2}\int_{-\infty}^{0} e^u\,du = \frac{1}{2}e^u\Big|_{-\infty}^{0} = \lim_{R\to-\infty} \frac{1}{2}e^u\Big|_R^0 = \frac{1}{2} - \frac{1}{2}\lim_{R\to-\infty} e^R = \frac{1}{2}$$

2. $\boxed{3}$ Using the comparison theorem, show the following integral converges.

$$\int_0^2 \frac{dx}{\sqrt{x}(2+x)}$$

Since $0 \le \frac{1}{\sqrt{x}(2+x)} \le \frac{1}{\sqrt{x}}$ & $\int_0^2 \frac{dx}{\sqrt{x}}$ is a convergent p-integral ($p = \frac{1}{2} < 1$),

by comparison $\int_0^2 \frac{dx}{\sqrt{x}(2+x)}$ also converges.

3. Consider the function $p(x) = \dfrac{C}{\sqrt{1-x^2}}$.

(a) $\boxed{3}$ Find C so that $p(t)$ is a probability density function on $[0, 1)$.

$$1 = \int_0^1 \frac{C}{\sqrt{1-x^2}}\,dx = C \arcsin x\Big|_0^1 = C\left(\frac{\pi}{2} - 0\right) = C\cdot\frac{\pi}{2}$$

so $C = \frac{2}{\pi}$

(b) $\boxed{1}$ Using the probability density function you found above, compute $P\left(0 \le X \le \frac{1}{2}\right)$.

$$P\left(0 \le X \le \tfrac{1}{2}\right) = \int_0^{1/2} \frac{2/\pi}{\sqrt{1-x^2}}\,dx = \frac{2}{\pi}\arcsin x\Big|_0^{1/2} = \frac{2}{\pi}\left(\frac{\pi}{6}\right) = \frac{1}{3}$$