Math 172 | Thing 4.4 | Name: _____

4 Apr 2017 | Show Appropriate Work | Point Values in boxes.

1. [2] Consider the power series $\sum_{n=0}^{\infty} b_n(x-3)^n$. Assume the series converges for $x=5$ and diverges for $x=0$. For each of the following values of x, determine if the series **C**onverges, **D**iverges, or if there is **N**ot enough information to tell.

   (a) **C** / (**D**) / **N** : $x=-1$

   (b) (**C**) / **D** / **N** : $x=2$

   (c) **C** / **D** / (**N**) : $x=6$

   (d) **C** / (**D**) / **N** : $x=8$

2. [2] Assume $\sqrt[n]{c_n} \to 2$ as $n \to \infty$. Find the radius of convergence R for the power series $\sum c_n(x+7)^n$.

   $R = \frac{1}{2}$

3. [2] What is the Taylor series about $x=c$ for $f(x)$.

   $$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

4. [2] Yesterday we showed the Taylor series about $x=0$ for $f(x)=e^x$ is $\sum_{n=0}^{\infty} \frac{x^n}{n!}$. Use the Ratio Test to find the radius of convergence R.

   $$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n}\right| = \frac{|x|}{n+1} \xrightarrow{n\to\infty} 0 \quad \text{so} \quad R=\infty$$

5. [2] Given $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x|<1$.

   (a) Find a power series representation for $\frac{1}{1+4x^2}$.

   $$\sum_{n=0}^{\infty} (-4x^2)^n$$

   (b) Find the interval of convergence for the series above.

   $|-4x^2| < 1$ so $|x| < \frac{1}{2}$

   i.e. $(-\frac{1}{2}, \frac{1}{2})$