

1. [4] Show the following integral converges or diverges using the Comparison Test.

$$\int_0^7 \frac{e^x}{x^2} dx$$

For  $0 < x \leq 7$ ,  $0 < \frac{1}{x^2} < \frac{e^x}{x^2}$ .

We know  $\int_0^7 \frac{dx}{x^2}$  is a divergent p-integral ( $p=2 > 1$ ),

so by comparison  $\int_0^7 \frac{e^x}{x^2} dx$  also diverges.

2. [3] Find the arc length of the graph of  $y = \frac{1}{3}(x^2 + 2)^{3/2}$  for  $0 \leq x \leq 3$ .

$$\begin{aligned} y' &= \frac{1}{3} \cdot \frac{2}{3} (x^2 + 2)^{\frac{1}{2}} (2x) & ds &= (x^2 + 1) dx \\ &= x \sqrt{x^2 + 2} \\ |+y'|^2 &= 1 + x^2 (x^2 + 2) & \text{so } s &= \int_0^3 (x^2 + 1) dx = \frac{x^3}{3} + x \Big|_0^3 = 12 \\ &= x^4 + 2x^2 + 1 \\ &= (x^2 + 1)^2 \end{aligned}$$

3. [3] A surface is generated by rotating the graph of  $y = e^x$  about the  $x$ -axis for  $0 \leq x \leq \ln(\sqrt{3})$ ; find the surface area. You will find the following helpful:

$$\begin{aligned} \int \sec^3 \theta d\theta &= \frac{1}{2} (\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta|) + C & \text{and} & \tan(\pi/3) = \sqrt{3} \\ S &= 2\pi \int_0^{\ln \sqrt{3}} e^x \sqrt{1+e^{2x}} dx = 2\pi \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^3 \theta d\theta = \pi \left( \sec \theta \tan \theta + \ln |\sec \theta + \tan \theta| \right) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} \\ \text{Let } e^x &= \tan \theta \\ e^x dx &= \sec^2 \theta d\theta \\ x=0 &\mapsto \theta = \frac{\pi}{4} \\ x=\ln \sqrt{3} &\mapsto \theta = \frac{\pi}{3} \end{aligned}$$