

You will find the following identities useful.

$$\cos 2x = \cos^2 x - \sin^2 x$$

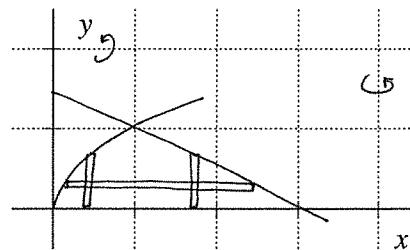
$$\tan^2 x + 1 = \sec^2 x$$

$$\int \sec u \, du = \ln |\sec u + \tan u| + c$$

1. Consider the region bounded by  $y = \sqrt{x}$ ,  $y = (3-x)/2$ , and the  $x$ -axis. Carefully sketch the region.

- (a) 4 The region is rotated about  $y = 2$ . Express the volume as a sum of two integrals. Do not evaluate.

$$\pi \int_0^1 ((2)^2 - (2 - \sqrt{x})^2) \, dx \\ + \pi \int_{\frac{1}{2}}^{\frac{3}{2}} ((2)^2 - (2 - \frac{3-x}{2})^2) \, dx$$



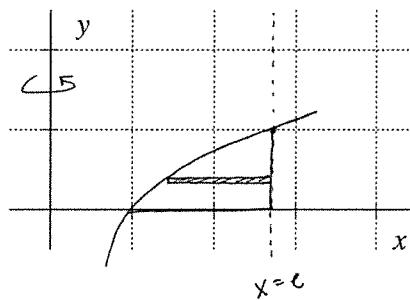
- (b) 6 The region is rotated about  $x = 4$ . Find the volume.

$$y = \frac{3-x}{2} \text{ so } 2y = 3-x \text{ or } x = 3 - 2y$$

$$\pi \int_0^1 [(4 - y^2)^2 - (4 - (3 - 2y))^2] \, dy = \pi \int_0^1 (16 - 8y^2 + y^4 - 1 - 4y + 4y^2) \, dy \\ = \pi \left[ 15y - 2y^3 - 4y^3 + \frac{y^5}{5} \right]_0^1 \\ = \pi \left( 9 + \frac{1}{5} \right) = \frac{46\pi}{5}$$

2. 10 Consider the region bounded by  $y = \ln x$ ,  $x = e$ , and the  $x$ -axis. Carefully sketch the region. The region is rotated about the  $y$ -axis, find the volume.

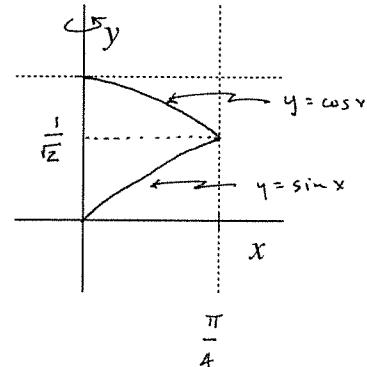
$$\pi \int_0^1 (e^y - e^{2y}) \, dy = \pi \left[ e^y - \frac{1}{2} e^{2y} \right]_0^1 \\ = \pi \left( e^2 - \frac{1}{2} e^2 + \frac{1}{2} \right) \\ = \frac{\pi}{2} (e^2 + 1)$$



3. Consider the region bounded by  $y = \cos x$  and  $y = \sin x$  for  $x \in [0, \pi/4]$ . Carefully sketch the region.

- (a) 4 The region is rotated about the  $y$ -axis. Express the volume as a sum of two integrals. Do not evaluate.

$$\pi \int_0^{\frac{1}{\sqrt{2}}} (\cos y)^2 dy + \pi \int_{\frac{1}{\sqrt{2}}}^1 (\sin y)^2 dy$$



- (b) 6 The region is rotated about the  $x$ -axis. Find the volume.

$$\pi \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x) dx = \pi \int_0^{\frac{\pi}{4}} \cos 2x dx = \frac{\pi}{2} \sin 2x \Big|_0^{\frac{\pi}{4}} = \frac{\pi}{2}$$

4. 10 Consider the region bounded by  $y = \tan x$  and  $y = \sec x$  for  $x \in [0, \pi/4]$ . Carefully sketch the region. The region is rotated about the line  $y = 2$ , find the volume.

$$\begin{aligned} & \pi \int_0^{\frac{\pi}{4}} [(2 - \tan x)^2 - (2 - \sec x)^2] dx \\ &= \pi \int_0^{\frac{\pi}{4}} [4 - 4 \tan x + \tan^2 x - 4 + 4 \sec x - \sec^2 x] dx \\ &= \pi \int_0^{\frac{\pi}{4}} (4 \sec x - 4 \tan x - 1) dx \\ &= \pi \left[ 4 \ln |\sec x + \tan x| - 4 \ln |\sec x| - x \right] \Big|_0^{\frac{\pi}{4}} \\ &= \pi \left( 4 \ln |\sqrt{2} + 1| - 4 \ln |\sqrt{2}| - \frac{\pi}{4} \right) \end{aligned}$$

