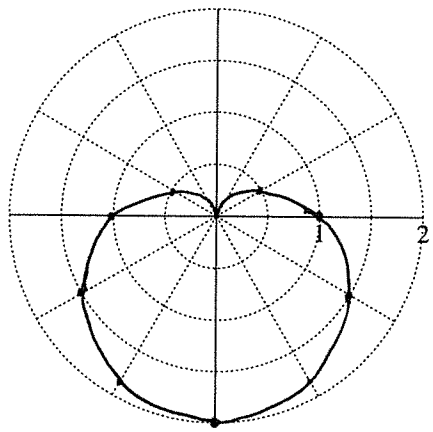
Math 172 | Quiz 11 | Name: _____

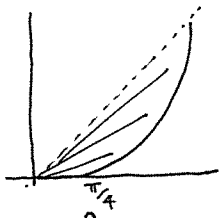
25 Apr 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

You may find the following identities useful: $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ and $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

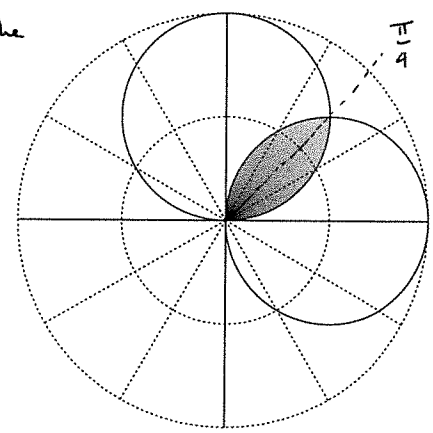
1. $\boxed{2}$ Carefully sketch the polar curve given by $r = 1 - \sin\theta$.



2. $\boxed{4}$ Find the area inside both $r = \sin\theta$ and $r = \cos\theta$, the shaded area in the figure.



By symmetry, we consider only the bottom half.



$$2\cdot\frac{1}{2}\int_0^{\pi/4} \sin^2\theta\, d\theta = \frac{1}{2}\int_0^{\pi/4}(1-\cos 2\theta)\,d\theta = \frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi/4}$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi - 2}{8}$$

3. $\boxed{4}$ Use an appropriate trigonometric substitution to integrate.

$$\int \frac{x^3}{\sqrt{1-x^2}}\,dx$$

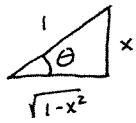
Let $x = \sin\theta$, $dx = \cos\theta\, d\theta$ so $1-x^2 = 1-\sin^2\theta = \cos^2\theta$

$$= \int \frac{\sin^3\theta}{\cos\theta}\cos\theta\, d\theta = \int \sin^3\theta\, d\theta = \int (1-\cos^2\theta)\sin\theta\, d\theta$$

Let $u = \cos\theta$, $du = -\sin\theta\, d\theta$

$$= \int (u^2-1)\,du = \frac{u^3}{3} - u + C = \frac{\cos^3\theta}{3} - \cos\theta + C$$

$$= \frac{1}{3}\left(\sqrt{1-x^2}\right)^3 - \sqrt{1-x^2} + C$$



so $\cos\theta = \sqrt{1-x^2}$