Math 172 — 27 Jan 2017

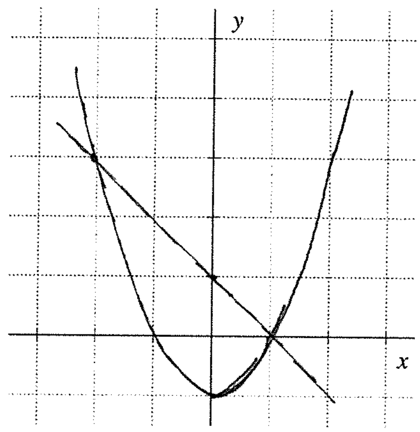
Quiz 2 — Show Appropriate Work

Name: ____________

Point Values in $\boxed{\text{boxes}}$.

1. Consider the region bounded by the graphs of $y = x^2 - 1$ and $y = 1 - x$.

   (a) $\boxed{1}$ Carefully sketch the graphs on the coordinate system below.



   (b) $\boxed{3}$ Find the area of the bounded region. [ProTip: In 172 'find the area' means compute an integral, not shade the area, draw an arrow to it, and exclaim "here it is!"]

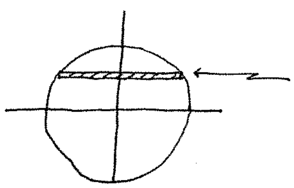
$$\int_{-2}^{1} \left(1-x - (x^2-1)\right)dx = \int_{-2}^{1} (2-x-x^2)\,dx = 2x - \frac{x^2}{2} - \frac{x^3}{3}\Big|_{-2}^{1}$$

$$= 2 - \frac{1}{2} - \frac{1}{3} - \left(-4 - 2 + \frac{8}{3}\right)$$

$$= 2 - \frac{1}{2} - \frac{1}{3} + 6 - \frac{8}{3}$$

$$= 5 - \frac{1}{2} = \frac{9}{2}$$

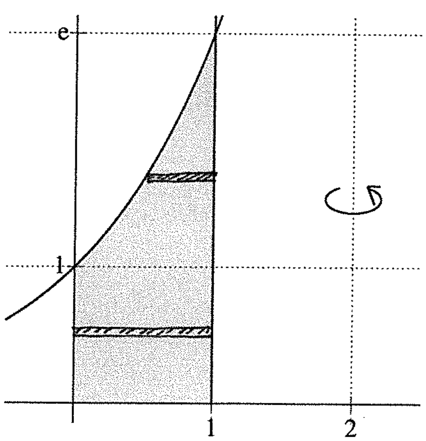
2. $\boxed{3}$ A solid is formed with base given by the unit circle $x^2+y^2 = 1$ and cross sections perpendicular to the y-axis are squares. Express the volume of the solid as an integral. **Do not evaluate the integral.**



$$V = (\text{side})^2 \Delta y$$
$$= (2x)^2 \Delta y$$
$$= 4x^2 \Delta y$$
$$= 4(1-y^2)\Delta y$$

$$V = \int_{-1}^{1} 4(1-y^2)\,dy$$

3. $\boxed{3}$ The region in the first quadrant bounded by the graphs of $y = e^x$ and $x = 1$, the shaded region in the figure, is revolved around the line $x = 2$. Express the volume of the resulting solid as an integral using the Disk Method. **Do not evaluate the integral.**



$$\pi \int_{0}^{1} (2^2 - 1^2)\,dy + \pi \int_{1}^{e} \left((2 - \ln y)^2 - 1^2\right) dy$$