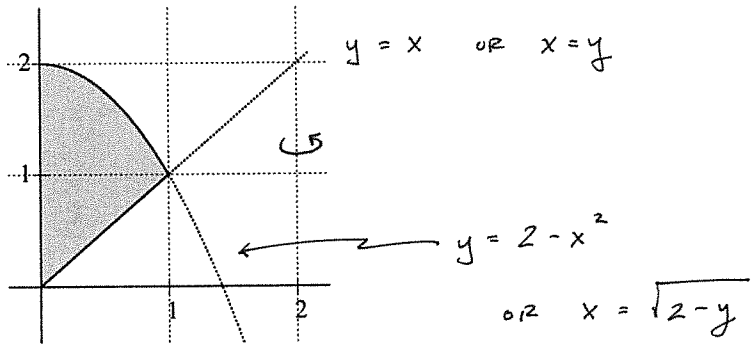
Math 172 **Name:** _____

Sections: 6.3-6.5

29 January 2019

1. Consider the region R bounded by the graphs of $y = x$, $y = 2 - x^2$, and the y-axis for $x \in [0, 1]$, the shaded region in the figure.



(a) The region is rotated about the x-axis. Express the volume using both the Disk Method and the Shell Method. **Do not evaluate the integral(s).**

$$V_{Disk} = \pi \int_0^1 \left[(2 - x^2)^2 - x^2 \right] dx$$

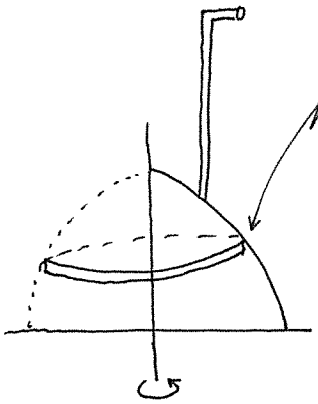
$$V_{Shell} = 2\pi \int_0^1 (y)(y)\, dy + 2\pi \int_1^2 (y)(\sqrt{2-y})\, dy$$

(b) The region is rotated about the line $x = 2$. Express the volume using both the Disk Method and the Shell Method. **Do not evaluate the integral(s).**

$$V_{Disk} = \pi \int_0^1 \left(2^2 - (2-y)^2 \right) dy + \pi \int_1^2 \left(2^2 - (2 - \sqrt{2-y})^2 \right) dy$$

$$V_{Shell} = 2\pi \int_0^1 (2 - x)\left[(2 - x^2) - x \right] dx$$

2. A tank is in the shape of the graph of $y = \sqrt{1-x}$ for $x \in [0,1]$ rotated about the y-axis. The tank is full of a mystery fluid of density ρ and is pumped out a spout of length 1 m from the top of the tank. Express your solution as a definite integral; **do not evaluate the integral.**



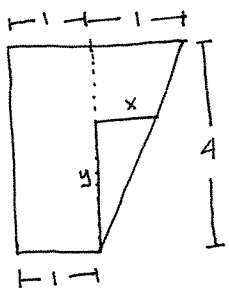
$$W_i = \pi (\text{radius})^2 \Delta y \cdot \rho \cdot g \cdot (2-y)$$

$$\text{radius} = x, \quad y = \sqrt{1-x} \quad \text{so} \quad y^2 = 1-x \quad \text{or} \quad x = 1-y^2$$

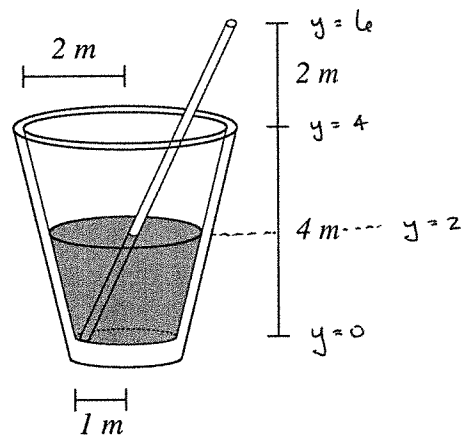
$$\text{Work} = \pi \rho g \int_0^1 (1-y^2)^2 (2-y)\, dy$$

3. After a long climb to the top of a beanstalk, Jack sees a giant drinking a murky red cocktail. The interior of the glass is in the shape of a frustum with lower radius 1 m and upper radius 2 m. The height of the interior of the glass is 4 m. The Blood of an Englishman[1] cocktail she is drinking is has a density of ρ. The giant is sipping her cocktail using a straw that extends 2 m past the top of the glass. When Jack arrived the giant had already consumed the 'top half' (as measured by height, not volume). How much work does the giant do by slurping the 'bottom half' up through the straw?

Express your solution as a definite integral; **do not evaluate the integral.** Please start by clearly identifying the coordinate system that you will be using.



$$\frac{x}{y} = \frac{1}{4} \quad \text{by similar } \Delta\text{'s}$$



$$\text{radius} = 1 + x = 1 + \frac{y}{4}$$

$$W_i = \pi (\text{radius}) \Delta y \cdot \rho \cdot g \cdot (6-y)$$

$$\text{Work} = \pi \rho g \int_0^2 \left(1 + \frac{y}{4}\right)^2 (6-y)\, dy$$

[1] No Englishmen were harmed in the writing of this question. No blood of any kind was used for her cocktail; the giant is a very pleasant vegan.