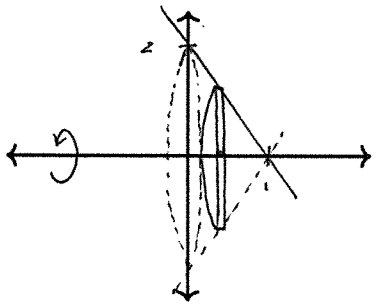
Math 172 In-Class **Name:** _____

Sections: 6.3

For each of the problems below, use the region bounded by $y = -2x + 2$, $x = 0$, and $y = 0$, and do the following:

1. Graph the rotated region.
2. Identify the variable you're integrating with respect to (w.r.t.).
3. State the radius/radii.
4. Set up the integral, and then find the volume.

1. Rotate: about the x-axis



Variable w.r.t.: x

Radius: $y = -2x + 2$

Area: $\pi(-2x+2)^2$

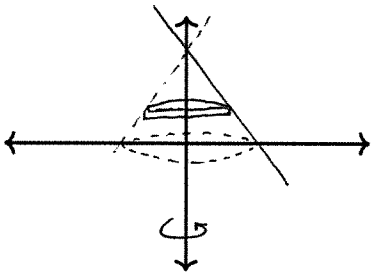
Volume:

$$\pi \int_0^1 4(1-x)^2\,dx = \pi \int_0^1 4u^2\,du = \frac{4\pi}{3}u^3\Big|_0^1 = \frac{4\pi}{3}$$

$u = 1-x \qquad 0 \longmapsto 1$

$du = -dx \qquad 1 \longmapsto 0$

2. Rotate: about the y-axis



Variable w.r.t.: y

Radius: $x = 1 - y/2$

Area: $\pi(1 - y/2)^2$

Volume:

$y - 2 = -2x$

$x = \frac{1}{2}(2-y)$

$= 1 - y/2$

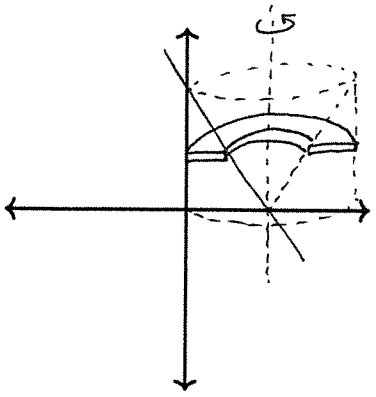
$$\pi \int_0^2 (1 - y/2)^2\,dy = \pi \int_0^2 (1 - y + y^2/4)\,dy$$

$$= \pi\left(y - y^2/2 + y^3/12\right)\Big|_0^2$$

$$= \pi\left(2 - 2 + \frac{8}{12}\right) = \frac{2\pi}{3}$$

CONTINUED ON THE REVERSE.

3. Rotate: about $x = 1$



Variable w.r.t.: y

Radius: $R_{out} = 1$, $r_{in} = 1 - x = 1 - (1 - y/2) = y/2$

Area: $\pi\left(1^2 - (y/2)^2\right) = \pi\left(1 - y^2/4\right)$

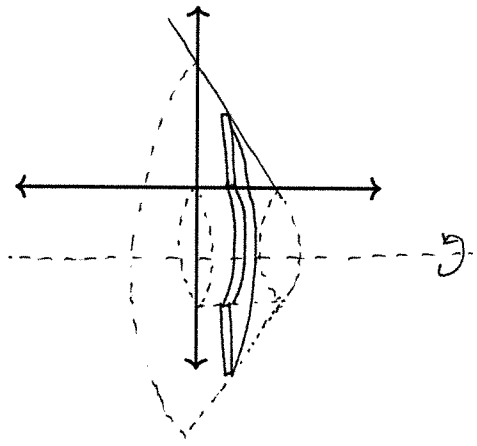
Volume:

$$\pi \int_0^2 \left(1 - \frac{y^2}{4}\right) dy = \pi \left(y - \frac{y^3}{12}\right)\Big|_0^2$$

$$= \pi\left(2 - \frac{8}{12}\right)$$

$$= \frac{4\pi}{3}$$

4. Rotate: about $y = -1$



Variable w.r.t.: x

Radius: $R_{out} = y + 1 = 3 - 2x$, $r_{in} = 1$

Area: $\pi\left((3-2x)^2 - 1\right) = \pi\left(9 - 12x + 4x^2 - 1\right)$

Volume:

$$\pi \int_0^1 \left(4x^2 - 12x + 8\right) dx$$

$$= 4\pi \int_0^1 \left(x^2 - 3x + 2\right) dx$$

$$= 4\pi \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)\Big|_0^1$$

$$= 4\pi \left(\frac{1}{3} - \frac{3}{2} + 2\right)$$

$$= \frac{10\pi}{3}$$