Math 172 Quiz 10 **Name:** _____

Sections: 10.3-10.4 Point values in $\boxed{\text{boxes}}$.

25 March 2019

1. $\boxed{1}$ For $c \neq 0$, the Geometric Series $\sum_{n=0}^{\infty} cr^n$ converges to $\frac{c}{1-r}$ for $|r| < 1$ and diverges for $|r| \geq 1$.

2. $\boxed{1}$ The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \leq 1$.

3. $\boxed{1}$ To use the Integral Test on the series $\sum_{n=1}^{\infty} a_n$, a function $f(x)$ with $f(n) = a_n$ must be positive, continuous, and decreasing.

4. $\boxed{3}$ For the series below, show the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty} \frac{\cos 4n}{(n+1)^2}$$

Since $0 < \left| \frac{\cos 4n}{(n+1)^2} \right| < \frac{1}{n^2}$

and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series $(p=2 > 1)$,

by comparison $\sum_{n=1}^{\infty} \left| \frac{\cos 4n}{(n+1)^2} \right|$ converges

and hence $\sum_{n=1}^{\infty} \frac{\cos 4n}{(n+1)^2}$ converges absolutely.

CONTINUED ON THE REVERSE.

5. $\boxed{4}$ For the series below, show the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty} \frac{\cos \pi n}{2n+2}$$

Initially we note $\cos \pi n = (-1)^n$

Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+2}$ converge absolutely?

Since $\left|\frac{(-1)^n}{2n+2}\right|$ and $\frac{1}{n}$ are both positive

& $\displaystyle\lim_{n\to\infty} \left|\frac{(-1)^n}{2n+2}\right| \Big/ \frac{1}{n} = \lim_{n\to\infty} \frac{n}{2n+2} = \frac{1}{2}$

which is positive and finite, by the Limit comparison test

$\displaystyle\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{2n+2}\right|$ diverges since the Harmonic series diverges.

Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+2}$ converge?

Since $a_n = \frac{1}{2n+2} > 0$, $a_n = \frac{1}{2n+2} > \frac{1}{2(n+1)+2} = a_{n+1}$, and $a_n = \frac{1}{2n+2} \xrightarrow[n\to\infty]{} 0$

by the Alternating series test $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+2}$ converges.

Finally,

Since $\displaystyle\sum_{n=1}^{\infty} \frac{\cos \pi n}{2n+2}$ converges but not absolutely, it converges conditionally.