Math 172 Quiz 11
Sections: 10.5-6
5 April 2019

Name: _____

Point values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Please circle **True** or **False**, as appropriate.

   (a) **T** / **F** : In the Ratio Test, if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| \geq 1$, the series $\sum a_n$ diverges.

   (b) **T** / **F** : $\sum a_n x^n$, if $\lim_{n\to\infty}\left|\frac{a_{n+1}x^{n+1}}{a_n x^n}\right| = 5|x|$, the radius of convergence R is $1/5$.

   (c) **T** / **F** : $\sum a_n(x-c)^n$, if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, the radius of convergence is 0.

   (d) **T** / **F** : $\sum a_n(x-1)^n$ converges when $x=0$, it must converge when $x=2$.

2. $\boxed{3}$ For each of the following series, circle if they converge or diverge and circle the test you used. (No justification required.)

$\sum_{n=1}^{\infty}\left(\frac{-2}{n}\right)^n$	converges diverges	Divergence test Integral Test Root Test
$\sum_{n=1}^{\infty}(-1)^n \cos\frac{1}{n}$	converges diverges	Divergence test Alternating Series Test Ratio Test
$\sum_{n=1}^{\infty}\frac{n}{2n^2+1}$	converges diverges	Ratio Test Limit Comparison Test Divergence Test

CONTINUED ON REVERSE.

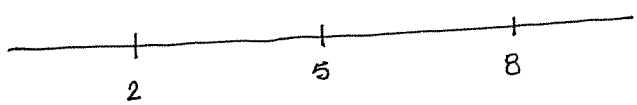
3. $\boxed{5}$ For the power series below, find and clearly state the radius and interval of convergence. **Justification required**: identify the tests you use, verify assumptions, and write conclusions.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-5)^n}{n\, 3^n}$$

$$\sqrt[n]{\left| \frac{(-1)^n (x-5)^n}{n\, 3^n} \right|} = \frac{|x-5|}{3} \cdot \frac{1}{\sqrt[n]{n}} \xrightarrow[n\to\infty]{} \frac{|x-5|}{3} < 1$$

so $|x-5| < 3$

The radius of convergence is $R=3$.



$x=2$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (-3)^n}{n\, 3^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ which is the divergent Harmonic series.

$x=8$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 3^n}{n\, 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ which is the <u>convergent</u> alternating Harmonic series.

The interval of convergence is $I = (2, 8]$.