Exercise 10.9.1. For each of the following series determine if they converge or diverge and then choose a test that can be used to show that.

1. $\sum \left(1+\frac{1}{n}\right)^{-n^2}$ Converges / Diverges by the Root Test / Divergence Test. $L = \frac{1}{e}$
2. $\sum \left(1+\frac{1}{n}\right)^{n}$ Converges / Diverges by the Root Test / Divergence Test.
3. $\sum \frac{1}{n \ln n}$ Converges / Diverges by the Ratio Test / Integral Test.
4. $\sum \frac{n+1}{n^2+2}$ Converges / Diverges by the LCT / Ratio Test.
5. $\sum \frac{(-1)^n}{n}$ Converges / Diverges by the Integral Test / AST.
6. $\sum (-n)^n$ Converges / Diverges by the Divergence Test / AST.

Exercise 10.9.2. For the following series, specify what series you would compare each to (either direct or limit comparison) and based on your comparison, decide if it converges or diverges. No formal justification is needed.

1. $\sum_{n=2}^{\infty} \frac{\sqrt{n^2+1}}{n^3+4n}$ compare to $\sum \frac{1}{n^2}$ so it CONVERGES / DIVERGES
2. $\sum_{n=2}^{\infty} \frac{3}{2^n\sqrt{n}}$ compare to $\sum \left(\frac{1}{2}\right)^n$ so it CONVERGES / DIVERGES
3. $\sum_{n=2}^{\infty} \frac{1}{n^2+\sqrt{n}}$ compare to $\sum \frac{1}{n^2}$ so it CONVERGES / DIVERGES
4. $\sum_{n=2}^{\infty} \frac{\sqrt{n^3+3}}{n^2+n}$ compare to $\sum \frac{1}{\sqrt{n}}$ so it CONVERGES / DIVERGES
5. $\sum_{n=2}^{\infty} \frac{2^n}{n3^n}$ compare to $\sum \left(\frac{2}{3}\right)^n$ so it CONVERGES / DIVERGES

Exercise 10.9.3. Choose all series below that...

1. can be shown to **converge** using the Divergence Test. NONE

   (a) $\sum \frac{4 \ln n}{n^2+1}$ (b) $\sum \frac{2}{n}$ (c) $\sum \frac{1}{e^n + n^2}$

2. can be shown to **diverge** using the Divergence Test.

   (a) $\sum \frac{1}{n}$ (b) $\sum \frac{n-1}{n}$ (c) $\sum \frac{1}{\ln n}$

3. can be shown to **converge** using either Comparison Test. NONE

   (a) $\sum \frac{1}{n \ln n}$ (b) $\sum \frac{(-1)^n}{n^2+1}$ (c) $\sum \frac{\ln n}{n}$

4. can be shown to **converge** using the Alternating Series Test.

   (a) $\sum \frac{(-1)^n}{\sqrt{n}}$ (b) $\sum \frac{\cos(\pi n)}{n}$ (c) $\sum \frac{(-1)^n n}{n+1}$

5. can be shown to **diverge** using the Alternating Series Test. NONE

   (a) $\sum \frac{\cos n}{n}$ (b) $\sum \frac{(-1)^n}{n}$ (c) $\sum \frac{(-1)^n n}{n+1}$

6. can be shown to **converge** using the Ratio or Root Test.

   (a) $\sum \frac{n}{n^3+5n}$ (b) $\sum \frac{3^n}{2^{2n+1}+1}$ (c) $\sum \frac{2^n}{n!}$

7. can be shown to **diverge** using the Ratio or Root Test.

   (a) $\sum \frac{n^2}{n^3+5n}$ (b) $\sum \frac{3^n}{2^{n+1}+1}$ (c) $\sum \frac{n^n}{n!}$

Exercise 10.9.4. For each of the following power series, determine the interval of convergence. Work through only as many of the details as needed to convince yourself that you are correct, the goal is to be able to identify the interval without a page worth of writing.

1. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$	$\lvert x-2\rvert < 1$	$[1, 3)$
2. $\sum_{n=1}^{\infty} \frac{2^n(x-2)^n}{n}$	$\lvert x-2\rvert < \frac{1}{2}$	$[3/2, 5/2)$
3. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n2^n}$	$\lvert x-2\rvert < 2$	$[0, 4)$
4. $\sum_{n=1}^{\infty} \frac{(-1)^n(x-2)^n}{n}$	$\lvert x-2\rvert < 1$	$(1, 3]$
5. $\sum_{n=1}^{\infty} \frac{x^n}{n^2+1}$	$\lvert x\rvert < 1$	$[-1, 1]$
6. $\sum_{n=1}^{\infty} \frac{nx^n}{n!}$		$(-\infty, \infty)$
7. $\sum_{n=1}^{\infty} \frac{2^n x^n}{n!}$		$(-\infty, \infty)$
8. $\sum_{n=1}^{\infty} \frac{\sqrt{n}(x+3)^n}{n^2+1}$	$\lvert x+3\rvert < 1$	$[-4, -2]$
9. $\sum_{n=1}^{\infty} \frac{(x+3)^n}{\sqrt{n}}$	$\lvert x+3\rvert < 1$	$[-4, -2)$
10. $\sum_{n=1}^{\infty} n(x-4)^n$	$\lvert x-4\rvert < 1$	$(3, 5)$