10.10.6

1. $$\frac{4x}{2+x} = 2x\left(\frac{1}{1-\left(\frac{-x}{2}\right)}\right) = 2x\sum_{n=0}^{\infty}\left(-\frac{x}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} 2^{1-n}(-1)^n x^{n+1} \quad \text{for } \left|\frac{x}{2}\right|<1 \quad \text{i.e. } |x|<2$$

2. $$\frac{\sin 2x}{x} = \sum_{n=0}^{\infty}\frac{(-1)^n\, 2^{2n+1}\, x^{2n}}{(2n+1)!} \quad \text{for all } x$$

3. $$xe^{x^2} = \sum_{n=0}^{\infty}\frac{x^{2n+1}}{n!} \quad \text{for all } x$$

4. $$\frac{5}{1+x^2} = \sum_{n=0}^{\infty} 5(-x^2)^n \quad \text{for } |x|<1$$

$$= \sum_{n=0}^{\infty} 5(-1)^n x^{2n}$$

5. $$\sin 3x = \sum_{n=0}^{\infty}\frac{(-1)^n (3x)^{2n+1}}{(2n+1)!} \quad \text{for all } x$$

6. $$e^{x+1} = e^x\cdot e = \sum_{n=0}^{\infty}\frac{e x^n}{n!} \quad \text{for all } x$$

10.10.7

1. $f(x) = (5-x)^{3/2} \quad c=1$

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	$(5-x)^{3/2}$	8
1	$3/2\,(5-x)^{1/2}(-1)$	-3
2	$3/2 \cdot 1/2\,(5-x)^{-1/2}$	$3/8$
3	$-3/8\,(5-x)^{-3/2}(-1)$	$3/64$

$$f(x) = 8 - 3(x-1) + \frac{3}{8 \cdot 2!}(x-1)^2 + \frac{3}{64 \cdot 3!}(x-1)^3 + \cdots$$

2. $g(x) = \sin 2x \quad c = \pi/4$

n	$f^{(n)}(x)$	$f^{(n)}(\frac{\pi}{4})$
0	$\sin 2x$	1
1	$2\cos 2x$	0
2	$-4\sin 2x$	-4
3	$-8\cos 2x$	0
	$\vdots$	

$$g(x) = 1 - \frac{4}{2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{16}{4!}\left(x - \frac{\pi}{4}\right)^4 - \frac{64}{6!}\left(x - \frac{\pi}{4}\right)^6 + \cdots$$

10.10.8

1. $$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x \to 0} \frac{x^2/2 + x^3/3! + \cdots}{x^2} = \frac{1}{2}$$

2. $$\lim_{x \to 0} \frac{x - \sin x}{x^3 \cos x} = \lim_{x \to 0} \frac{+ x^3/3! - x^5/5! - \cdots}{x^3 - x^5/2! + \cdots} = \frac{1}{3!}$$

10.10.9

1. $$\int e^{x^4}\,dx = \int \sum_{n=0}^{\infty} \frac{x^{4n}}{n!}\,dx$$

$$= A + \sum_{n=0}^{\infty} \frac{x^{4n+1}}{(4n+1)\,n!}$$

2. $$\int \sin x^2\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{4n+2}}{(2n+1)!}\,dx$$

$$= A + \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{4n+3}}{(4n+3)(2n+1)!}$$

10.10.10

1. $\cos 2\pi = 1$

2. $e^{\ln 2} = 2$