10.5.4.

1. False

2. True

3. False

10.5.5.

$C = -1$, $R = 3$

10.5.6

1. $\displaystyle\sum_{n=0}^{\infty} (-4)^n x^n$ for $|x| < \frac{1}{4}$

2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{8^{n+1}}$ for $|x| < 2$

3. $\displaystyle\sum_{n=0}^{\infty} x^{n+1}$ for $|x| < 1$

4. $\displaystyle\sum_{n=0}^{\infty} \frac{3(-1)^n x^{4n+2}}{2^{n+1}}$ for $|x| < \sqrt[4]{2}$

5. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-4)^n}{(-3)^{n+1}}$ for $|x-4| < 3$

6. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{4^n}$ for $|x-2| < 4$

7. $\displaystyle\sum_{n=1}^{\infty} n(-1)^{n+1} x^{n-1}$ for $|x| < 1$

8. $\displaystyle\sum_{n=2}^{\infty} 2n(n-1)(-1)^n x^{n-2}$

for $|x| < 1$

10.5.7

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{so}$$

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \quad \text{so}$$

$$\int e^{x^2} dx = A + \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)n!}$$

$$= A + \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)n!}$$

(for all x)

1. $\dfrac{1}{1+4x} = \dfrac{1}{1-(-4x)} = \displaystyle\sum_{n=0}^{\infty} (-4x)^n = \sum_{n=0}^{\infty} (-4)^n x^n$

for $|-4x|<1$ i.e. $|x| < \dfrac{1}{4}$

2. $\dfrac{1}{8+x^3} = \dfrac{1/8}{1-\left(\frac{-x^3}{8}\right)} = \displaystyle\sum_{n=0}^{\infty} \frac{1}{8}\left(-\frac{x^3}{8}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{8^{n+1}}$

for $\left|-\dfrac{x^3}{8}\right| < 1$ i.e. $|x| < 2$

3. $\dfrac{x}{1-x} = x \cdot \dfrac{1}{1-x} = x \displaystyle\sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} x^{n+1}$

for $|x|<1$

4. $\dfrac{3x^2}{2+x^4} = \dfrac{3x^2}{2} \cdot \dfrac{1}{1-\left(\frac{-x^4}{2}\right)} = \dfrac{3x^2}{2} \displaystyle\sum_{n=0}^{\infty} \left(\frac{-x^4}{2}\right)^n = \sum_{n=0}^{\infty} \frac{3(-1)^n x^{4n+2}}{2^{n+1}}$

for $|x| < \sqrt[4]{2}$

5. $\dfrac{1}{1-x} = \dfrac{1}{1-(x-4)-4} = \dfrac{1}{-3-(x-4)} = \dfrac{-1/3}{1-\left(\frac{x-4}{-3}\right)}$

$= -\dfrac{1}{3} \displaystyle\sum_{n=0}^{\infty} \left(\frac{x-4}{-3}\right)^n = \sum_{n=0}^{\infty} \frac{(x-4)^n}{(-3)^{n+1}}$ for $|x-4| < 3$

6. $\dfrac{4}{2+x} = \dfrac{4}{2+(x-2)+2} = \dfrac{4}{4+(x-2)} = \dfrac{1}{1+\left(\frac{x-2}{4}\right)} = \displaystyle\sum_{n=0}^{\infty} \left(\frac{x-2}{4}\right)^n$ $|x-2| < 4$

7. $$\frac{1}{(1+x)^2} = \frac{d}{dx}\left[\frac{-1}{(1+x)}\right] = -1\,\frac{d}{dx}\sum_{n=0}^{\infty}(-1)^n x^n$$

$$= -1\sum_{n=1}^{\infty} n(-1)^n x^{n-1} = \sum_{n=1}^{\infty} n(-1)^{n+1}x^{n-1} \quad \text{for } |x|<1$$

8. $$\frac{1}{(1+x)^3} = \frac{d}{dx}\left[\frac{-2}{(1+x)^2}\right] = -2\frac{d}{dx}\sum_{n=1}^{\infty} n(-1)^{n+1}x^{n-1}$$

$$= -2\sum_{n=2}^{\infty} n(n-1)(-1)^{n+1}x^{n-2}$$

$(-1)^n = (-1)^{n+2}$

$$= \sum_{n=2}^{\infty} 2n(n-1)(-1)^n x^{n-2} \quad \text{for } |x|<1$$