Math 172 | Practice Exam 1 | Name: ____________
Summer 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. Integrate.

(a) $\boxed{10}$ $\displaystyle\int_0^3 t^2\sqrt{1+t}\,dt = \int_1^4 (u-1)^2\sqrt{u}\,du = \int_1^4 \left(u^{5/2} - 2u^{3/2} + u^{1/2}\right) du$

Let $u = 1+t$

$du = dt$

t	$\longmapsto$	u
0		1
3		4

$$= \frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2}\Bigg|_1^4$$

$$= \frac{2}{7}(128-1) - \frac{4}{5}(32-1) + \frac{2}{3}(8-1)$$

$$= \frac{2}{7}(127) - \frac{4}{5}(31) + \frac{2}{3}(7)$$ ← reasonably simplified

(b) $\boxed{10}$ $\displaystyle\int x\arctan x\,dx = \frac{x^2}{2}\arctan x - \int \frac{x^2/2}{x^2+1}\,dx = \frac{x^2}{2}\arctan x - \frac{1}{2}\int\left(\cancel{\frac{x^2+1}{x^2+1}}^{\,1} - \frac{1}{x^2+1}\right)dx$

$u = \arctan x$ $\quad dv = x\,dx$

$du = \dfrac{dx}{1+x^2}$ $\quad v = \dfrac{x^2}{2}$

$$= \frac{x^2}{2}\arctan x - \frac{1}{2}\Big[x - \arctan x\Big] + C$$

(c) $\boxed{10}$ $\displaystyle\int_0^{\pi/2} \sin^3 2\theta\,d\theta = \int_0^{\pi/2}(1-\cos^2 2\theta)\sin 2\theta\,d\theta = -\frac{1}{2}\int_1^{-1}(1-u^2)\,du$

$u = \cos 2\theta$

$du = -2\sin 2\theta\,d\theta$

θ	$\longmapsto$	u
0		1
$\pi/2$		-1

$$= \frac{1}{2}\left(u - \frac{u^3}{3}\right)\Bigg|_{-1}^{1} = u - \frac{u^3}{3}\Bigg|_0^1 = \frac{2}{3}$$

2. Integrate.

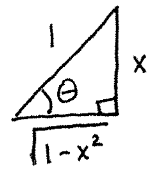
(a) $\boxed{10}$ $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^3\theta}{\cos\theta}\cdot\cos\theta\,d\theta = \int (1-\cos^2\theta)\sin\theta\,d\theta$

Let $x = \sin\theta$

$dx = \cos\theta\,d\theta$

so $1-x^2 = 1-\sin^2\theta$

$= \cos^2\theta$



$u = \cos\theta$

$du = -\sin\theta\,d\theta$

$$= \int (u^2-1)\,du = \frac{u^3}{3} - u + C$$

$$= \frac{\cos^3\theta}{3} - \cos\theta + C$$

$$= \frac{1}{3}(1-x^2)^{3/2} - \sqrt{1-x^2} + C$$

(b) $\boxed{10}$ $\displaystyle\int \frac{4x}{x^2-4x+8}\,dx = \int \frac{4x-8}{x^2-4x+8}\,dx + \int \frac{8}{(x-2)^2+2^2}\,dx$

$$= 2\ln(x^2-4x+8) + 4\arctan\left(\frac{x-2}{2}\right) + C$$

(c) $\boxed{10}$ $\displaystyle\int \frac{4x^2+5x+3}{x(x+1)^2}\,dx = \int\left(\frac{3}{x} + \frac{1}{x+1} - \frac{2}{(x+1)^2}\right)dx = 3\ln|x| + \ln|x+1| + \frac{2}{x+1} + C$

$$\frac{4x^2+5x+3}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

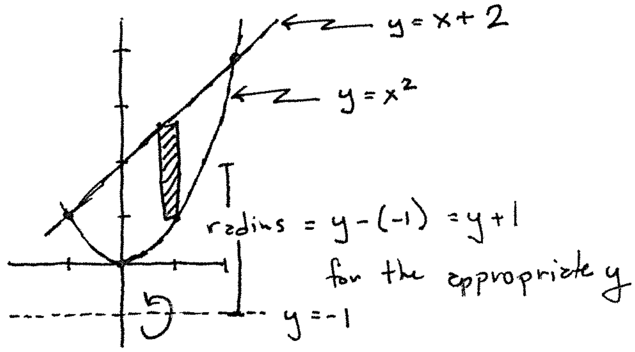
$$4x^2+5x+3 = A(x+1)^2 + Bx(x+1) + Cx$$

let $x=0$, $3 = A$

Equate coeff: x^2: $4 = A + B \Rightarrow B = 1$

x : $5 = 2A + B + C \Rightarrow C = -2$

3. $\boxed{10}$ Consider the region bounded by the graphs of $y = x^2$ and $y = x + 2$. Carefully sketch the region. Find the volume of the solid generated by rotating the region about the line $y = -1$.



Shells would require two integrals, we should use washers.

Outer Radius $= x + 2 + 1 = x + 3$

inner radius $= x^2 + 1$

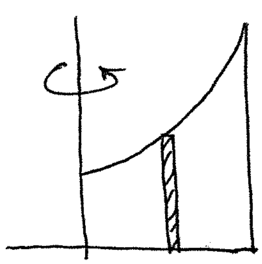
$$V = \int_{-1}^{2} \pi\left((x+3)^2 - (x^2+1)^2\right) dx = \pi \int_{-1}^{2} \left(8 + 6x - x^2 - x^4\right) dx = \pi\left(8x + 3x^2 - \frac{x^3}{3} - \frac{x^5}{5}\right)\Bigg|_{-1}^{2}$$

Any of these are "reasonable"

$$= \pi\left[\left(16 + 12 - \frac{8}{3} - \frac{32}{5}\right) - \left(-8 + 3 + \frac{1}{3} + \frac{1}{5}\right)\right] = \pi\left[33 - \frac{9}{3} - \frac{33}{5}\right]$$

$$= \pi\left(30 - \frac{33}{5}\right) = \pi\left(\frac{117}{5}\right)$$

4. $\boxed{10}$ Consider the region between the graph of $y = e^x$ and the x-axis for $x \in [0, 1]$. Carefully sketch the region. Find the volume of the solid generated by rotating the region about the y-axis.



Using washers requires two integrals, we should use shells.

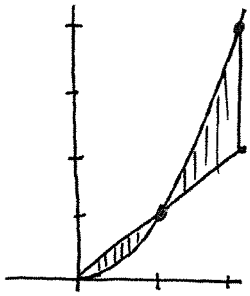
$$V = 2\pi \int_0^1 x e^x \, dx = 2\pi\left(x e^x\Big|_0^1 - \int_0^1 e^x \, dx\right) = 2\pi\left(e - e^x\Big|_0^1\right)$$

$u = x \qquad dv = e^x \, dx$

$du = dx \qquad v = e^x$

$$= 2\pi$$

5. $\boxed{10}$ Consider the area between the graphs of the functions $y = x^2$ and $y = x$ for $x \in [0, 2]$. Carefully sketch the graphs and find the area.



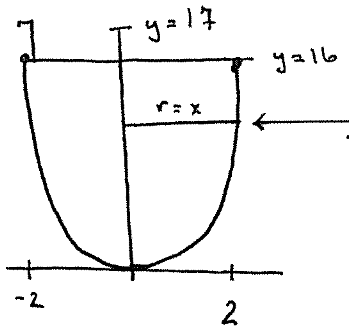
$$A = \int_0^1 (x - x^2)\,dx + \int_1^2 (x^2 - x)\,dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 + \left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_1^2$$

$$= \frac{1}{2} - \frac{1}{3} + \left(\frac{8}{3} - \frac{4}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)$$

$$= 1 - \frac{2}{3} + \frac{8}{3} - 2 = 1$$

6. $\boxed{10}$ Calculate the work (in joules) required to pump all of the mystery fluid of density ρ out of a full tank on the planet Zanabar with gravitational constant g. The tank is in the shape of the graph of $y = x^4$ for $x \in [0, 2]$ rotated about the y-axis. The mystery fluid is pumped out a spout of length 1 m from the top of the tank.



distance

$$W_i = \rho g \pi\, (\text{radius})^2\, (17 - y)\, \Delta y$$

$$= \rho g \pi\, x^2\, (17 - y)\, \Delta y \qquad \text{but } y = x^4$$

$$= \rho g \pi \sqrt{y}\, (17 - y)\, \Delta y \qquad \text{so } \sqrt{y} = x^2$$

$$\text{Work} = \rho g \pi \int_0^{16} \left(17\sqrt{y} - y^{3/2}\right) dy = \rho g \pi \left(\frac{34}{3} y^{3/2} - \frac{2}{5} y^{5/2}\right)\Big|_0^{16}$$

$$= \rho g \pi \left(\frac{34}{3} \cdot 16^{3/2} - \frac{2}{5} \cdot 16^{5/2}\right)$$