Solutions are again cobbled together from previous review material. They are not in the correct order.

Page	Solutions for
2	1. A-D
3	1. F-I
4	1. K-N
5	2. A-F
6	3. A-F
7	4. A-E
8	4. F, 5 A-B
9	5 C-E
10	5. F, 1. E, 6. A
11	1. J, 6. B, 1. O, 6. C

Work problems: See text & previously posted solutions

Math 172 Final Exam Review

1\. A. $$\int (x+3)\sin 2x\,dx = -\frac{(x+3)\cos 2x}{2} + \int \frac{\cos 2x}{2}\,dx = \frac{\sin 2x}{4} - \frac{(x+3)\cos 2x}{2} + C$$

$u = x+3 \qquad dv = \sin 2x\,dx$

$du = dx \qquad v = -\frac{\cos 2x}{2}$

B. $$\int \sin x \cos^7 x\,dx = -\int u^7\,du = -\frac{\cos^8 x}{8} + C$$

$u = \cos x$

$du = -\sin x\,dx$

C. $$\int \sqrt{9-x^2}\,dx = \int 9\cos^2\theta\,d\theta = \frac{9}{2}\int (1+\cos 2\theta)\,d\theta = \frac{9}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

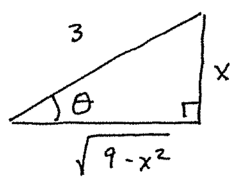
$\sin 2\theta = 2\sin\theta\cos\theta$

let $x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

So $9 - x^2 = 9 - 9\sin^2\theta$

$= 9\cos^2\theta$



$$= \frac{9}{2}\left(\theta + \sin\theta\cos\theta + C\right)$$

$$= \frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right) + \frac{x\sqrt{9-x^2}}{9}\right) + C$$

D. $$\int \frac{7x-6}{(2x+1)(x+3)}\,dx = \int \left(\frac{1}{2x+1} + \frac{3}{x+3}\right)dx = \frac{1}{2}\ln|2x+1| + 3\ln|x+3| + C$$

$$\frac{7x+6}{(2x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+3}$$

$$7x+6 = A(x+3) + B(2x+1)$$

let $x = -3$: $\quad -15 = B(-5) \Rightarrow B = 3$

$x = -\frac{1}{2}$: $\quad \frac{5}{2} = A\left(\frac{5}{2}\right) \Rightarrow A = 1$

F $$\int x^2 e^{-4x}\,dx = -\frac{1}{4}x^2e^{-4x} + \int \frac{1}{2}xe^{-4x}\,dx = -\frac{1}{4}x^2e^{-4x} - \frac{1}{8}xe^{-4x} + \int \frac{1}{8}e^{-4x}\,dx$$

$u = x^2 \qquad dv = e^{-4x}\,dx$

$du = 2x\,dx \qquad v = -\frac{1}{4}e^{-4x}$

$u = \frac{1}{2}x \qquad dv = e^{-4x}\,dx$

$du = \frac{1}{2}dx \qquad v = -\frac{1}{4}e^{-4x}$

$$= -\frac{1}{4}x^2e^{-4x} - \frac{1}{8}xe^{-4x} - \frac{1}{32}e^{-4x} + C$$

G $$\int \sec^4 4\theta \tan^4 4\theta\,d\theta = \int \underbrace{(\tan^2 4\theta + 1)}_{=\sec^2 4\theta} \tan^4 4\theta \sec^2 4\theta\,d\theta = \frac{1}{4}\int (u^2+1)\,u^4\,du$$

let $u = \tan 4\theta$

$du = 4\sec^2 4\theta\,d\theta$

$$= \frac{1}{4}\left(\frac{\tan^7 4\theta}{7} + \frac{\tan^5 4\theta}{5}\right) + C$$

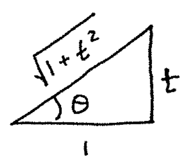
H $$\int \frac{dt}{t^4+2t^2+1} = \int \frac{dt}{(t^2+1)^2} = \int \frac{\sec^2\theta}{\sec^4\theta}\,d\theta = \int \cos^2\theta\,d\theta = \frac{1}{2}\int (1+\cos 2\theta)\,d\theta$$

Let $t = \tan\theta$

$dt = \sec^2\theta\,d\theta$

so $t^2+1 = \tan^2\theta + 1$

$= \sec^2\theta$



$\sin 2\theta = 2\cos\theta\sin\theta$

$$= \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{1}{2}\left(\theta + \sin\theta\cos\theta\right) + c$$

$$= \frac{1}{2}\left(\arctan t + \frac{(t)(1)}{(\sqrt{1+t^2})^2}\right) + c$$

$$= \frac{1}{2}\left(\arctan t + \frac{t}{1+t^2}\right) + c$$

I $$\int \frac{4x^2+3x+2}{x^3+x^2}\,dx = \int \left(\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x+1}\right)dx$$

$$= \ln|x| - \frac{2}{x} + 3\ln|x+1| + C$$

$$\frac{4x^2+3x+2}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$4x^2+3x+2 = Ax(x+1) + B(x+1) + Cx^2$$

Let $x=0$: $2 = B$

Let $x=-1$: $3 = C$

Equate coeff to find A

x^2: $4 = A + C \Rightarrow A = 1$

K. $\displaystyle\int_1^e t\ln t\,dt = \frac{1}{2}t^2\ln t\Big|_1^e - \int_1^e \frac{1}{2}t\,dt = \frac{1}{2}e^2 - \frac{1}{4}t^2\Big|_1^e = \frac{1}{2}e^2 - \frac{1}{4}e^2 + \frac{1}{4} = \frac{1}{4}\left(e^2+1\right)$

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$u = \ln t \qquad dv = t\,dt$

$du = \frac{1}{t}dt \qquad v = \frac{1}{2}t^2$

$\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ $\qquad$ $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

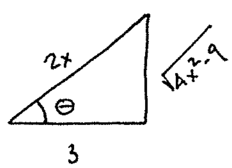
L. $\displaystyle\int \sin^2(\pi x)\cos^2(\pi x)\,dx = \frac{1}{4}\int \sin^2(2\pi x)\,dx = \frac{1}{8}\int\left(1-\cos(4\pi x)\right)dx$

$\displaystyle = \frac{1}{8}\left(x - \frac{1}{4\pi}\sin(4\pi x)\right) + C$

M. $\displaystyle\int \frac{\sqrt{4x^2-9}}{x}\,dx = \int \frac{3\tan\theta}{\frac{3}{2}\sec\theta}\cdot\frac{3}{2}\sec\theta\tan\theta\,d\theta = 3\int \tan^2\theta\,d\theta = 3\int\left(\sec^2\theta - 1\right)d\theta$

Let $2x = 3\sec\theta$

$2dx = 3\sec\theta\tan\theta\,d\theta$



$= 3\left(\tan\theta - \theta\right) + C$

so $4x^2 - 9 = 9\sec^2\theta - 9$

$= 9\tan^2\theta$

$\displaystyle = 3\left(\frac{\sqrt{4x^2-9}}{3} - \operatorname{arcsec}\left(\frac{2x}{3}\right)\right) + C$

N. $\displaystyle\int \frac{5x+6}{(x-2)(x^2+4)}\,dx = \int\left(\frac{2}{x-2} - \frac{2x}{x^2+4} + \frac{1}{x^2+2^2}\right)dx$

$u = x^2+4$

$du = 2x\,dx$

$\displaystyle\frac{5x+6}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4}$

$= 2\ln|x-2| - \ln(x^2+4) + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

$5x+6 = A(x^2+4) + (Bx+C)(x-2)$

Let $x=2$: $\quad 16 = A(8) \Rightarrow A = 2$

Note: $\displaystyle\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$

is given information.

Equat coeff: $\underline{x^2}$ $\qquad$ $\underline{x^0}$

$0 = A + B \qquad 6 = 4A - 2C$

so $B = -2 \qquad$ so $C = 1$

2.A. $\displaystyle\int x\cos 2x\,dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$

$u = x \qquad dv = \cos 2x\,dx$

$du = dx \qquad v = \frac{1}{2}\sin 2x$

B. $\displaystyle\int x\arctan x\,dx = \frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx = \frac{x^2}{2}\arctan x - \frac{1}{2}\int\left(\overset{=1}{\frac{x^2+1}{x^2+1}} - \frac{1}{x^2+1}\right)dx$

$u = \arctan x \qquad dv = x\,dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = \frac{x^2}{2}$

$\displaystyle = \frac{1}{2}\left(x^2\arctan x - x + \arctan x\right) + C$

C. $\displaystyle\int_0^{\pi/4} x\sec^2 x\,dx = x\tan x\Big|_0^{\pi/4} - \int_0^{\pi/4}\tan x\,dx = \frac{\pi}{4} - \ln\left|\sec x\right|\Big|_0^{\pi/4}$

$u = x \qquad dv = \sec^2 x\,dx$

$\displaystyle = \frac{\pi}{4} - \ln\sqrt{2}$

$du = dx \qquad v = \tan x$

D. $\displaystyle\int \ln x\,dx = x\ln x - \int dx = x\ln x - x + C$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

E. $\displaystyle\int x^2\sin 2x\,dx = -\frac{x^2}{2}\cos 2x + \int x\cos 2x\,dx = -\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos x + C$

$u = x^2 \qquad dv = \sin 2x\,dx$

$du = 2x\,dx \qquad v = -\frac{1}{2}\cos 2x$

$\left\{\begin{array}{ll} u = x & dv = \cos 2x\,dx \\ du = dx & v = \frac{1}{2}\sin 2x \end{array}\right.$

F. $\displaystyle\int x^5\ln x\,dx = \frac{1}{6}x^6\ln x - \frac{1}{36}x^6 + C$

$u = \ln x \qquad dv = x^5\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{6}x^6$

Exam 3 Review - Solutions

3. A. $\displaystyle\int_0^{\pi} \sin^3 5x\, dx = \int_0^{\pi} (1-\cos^2 5x)\sin 5x\, dx = -\frac{1}{5}\int_1^{-1} (1-u^2)\,du = \frac{1}{5}\int_{-1}^{1} (1-u^2)\,du$

let $u = \cos 5x$

$du = -5\sin 5x$

$x = 0 \longmapsto u = 1$

$x = \pi \longmapsto u = -1$

$$= -\frac{1}{5}\left(u - \frac{u^3}{3}\right)\Big|_{-1}^{1} = \frac{1}{5}\left[\left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right)\right] \leftarrow \text{Stop here}$$

$$= \frac{1}{5}\cdot\frac{4}{3} = \frac{4}{15}$$

B. $\displaystyle\int \sin^9 x \cos^3 x\, dx = \int \sin^9 x\,(1-\sin^2 x)\cos x\, dx = \int u^9(1-u^2)\,du$

let $u = \sin x$

$du = \cos x\, dx$

$$= \frac{\sin^{10} x}{10} - \frac{\sin^{12} x}{12} + C$$

C. $\displaystyle\int \sin^2 \pi x\, dx = \int \frac{1}{2}(1-\cos 2\pi x)\,dx = \frac{1}{2}\left(x - \frac{1}{2\pi}\sin 2\pi x\right) + C$

D. $\displaystyle\int \tan^2 x \sec^2 x\, dx = \frac{1}{3}\tan^3 x + C$

$u = \tan x \quad du = \sec^2 x\, dx$

E. $\displaystyle\int \tan^6 x \sec^4 x\, dx = \int \tan^6 x\,(1+\tan^2 x)\sec^2 x\, dx = \int u^6(1+u^2)\,du$

$u = \tan x \quad du = \sec^2 x\, dx$

$$= \frac{\tan^7 x}{7} + \frac{\tan^9 x}{9} + C$$

F. $\displaystyle\int \sec^3 2x \tan^3 2x\, dx = \int \sec^2 2x\,(\sec^2 2x - 1)\sec 2x \tan 2x\, dx$

$u = \sec 2x \quad du = 2\sec 2x \tan 2x\, dx$

$$= \frac{1}{2}\int u^2(u^2-1)\,du = \frac{1}{2}\int (u^4 - u^2)\,du$$

$$= \frac{\sec^5 2x}{10} - \frac{\sec^3 2x}{6} + C$$

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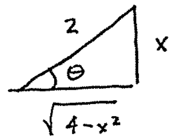
4. A. $\int \sqrt{4-x^2}\,dx = \int 2\cos\theta \cdot 2\cos\theta \, d\theta = \int 2(1+\cos 2\theta)\,d\theta$

Given: $\sin 2\theta = 2\sin\theta\cos\theta$

Let $x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$



$= 2\left(\theta + \frac{1}{2}\sin 2\theta\right) + C = 2(\theta + \sin\theta\cos\theta) + C$

$= 2\left(\arcsin\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{4}\right) + C$

B. $\int_{\sqrt{3}}^{2} \frac{\sqrt{x^2-3}}{x}\,dx = \int_0^{\pi/6} \frac{\sqrt{3}\tan\theta}{\sqrt{3}\sec\theta}\sqrt{3}\sec\theta\tan\theta\,d\theta = \sqrt{3}\int_0^{\pi/6} \tan^2\theta\,d\theta$

Let $x = \sqrt{3}\sec\theta$

$dx = \sqrt{3}\sec\theta\tan\theta\,d\theta$

$x = \sqrt{3} \mapsto \theta = 0$

$x = 2 \mapsto \theta = \pi/6$

$= \sqrt{3}\int_0^{\pi/6} (\sec^2\theta - 1)\,d\theta$

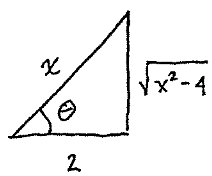
$= \sqrt{3}\left(\tan\theta - \theta\right)\Big|_0^{\pi/6} = \sqrt{3}\left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right)$

C. $\int \frac{x^2}{\sqrt{x^2-4}}\,dx = \int \frac{4\sec^2\theta}{\cancel{2\tan\theta}} \cdot \cancel{2}\sec\theta\,\cancel{\tan\theta}\,d\theta = 4\int \sec^3\theta\,d\theta$

This is a given integral

Let $x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$



$= 2\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right) + C$

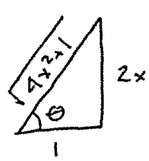
$= 2\left(\frac{x}{2}\cdot\frac{\sqrt{x^2-4}}{2} + \ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right|\right) + C$

D. $\int \frac{dx}{\sqrt{4x^2+1}} = \frac{1}{2}\int \frac{\sec^2\theta\,d\theta}{\sec\theta} = \frac{1}{2}\int \sec\theta\,d\theta = \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

given

Let $2x = \tan\theta$

$2\,dx = \sec^2\theta\,d\theta$

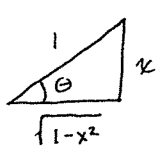


$= \frac{1}{2}\ln\left|\sqrt{4x^2+1} + 2x\right| + C$

E. $\int \frac{x^3}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^3\theta}{\cancel{\cos\theta}}\cdot\cancel{\cos\theta}\,d\theta = \int (1-\cos^2\theta)\sin\theta\,d\theta = -\int (1-u^2)\,du$

Let $x = \sin\theta$

$dx = \cos\theta\,d\theta$



$u = \cos\theta$

$du = -\sin\theta\,d\theta$

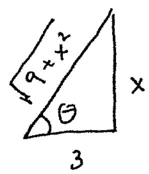
$= \frac{u^3}{3} - u + C = \frac{\cos^3\theta}{3} - \cos\theta + C$

$= \frac{(1-x^2)^{3/2}}{3} - \sqrt{1-x^2} + C$

4F. $\displaystyle\int \frac{9}{(9+x^2)^2}\,dx = \int \frac{\cancel{9}\cdot 3\sec^2\theta}{\cancel{9}\cdot 9\sec^4\theta}\,d\theta = \frac{1}{3}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{3}\int \cos^2\theta\,d\theta$

Let $x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$



$= \frac{1}{6}\int (1+\cos 2\theta)\,d\theta = \frac{1}{6}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$

$\sin 2\theta = 2\sin\theta\cos\theta$, given

$= \frac{1}{6}(\theta + \sin\theta\cos\theta) + C$

$= \frac{1}{6}\left(\arctan\left(\frac{x}{3}\right) + \frac{3x}{9+x^2}\right) + C$

5.A. $\displaystyle\int \frac{3-x}{1-x^2}\,dx = \int \left(\frac{1}{1-x} + \frac{2}{1+x}\right)dx$

$= -\ln|1-x| + 2\ln|1+x| + C$

$\frac{3-x}{(1-x)(1+x)} = \frac{A}{1-x} + \frac{B}{1+x}$

$3-x = A(1+x) + B(1-x)$

Let $x = 1$, $2 = A(2)$ so $A = 1$

Let $x = -1$, $4 = B(2)$ so $B = 2$

B. $\displaystyle\int_{-1}^{0} \frac{3}{x^2+x-2}\,dx = \int_{-1}^{0}\left(\frac{1}{x-1} - \frac{1}{x+2}\right)dx = \ln|x-1| - \ln|x+2|\Big|_{-1}^{0}$

$= \ln|-1| - \ln|2| - \ln|-2| + \ln|1|$

$= -2\ln(2)$

$\frac{3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

$3 = A(x+2) + B(x-1)$

Let $x = 1$, $3 = A(3)$ so $A = 1$

Let $x = -2$, $3 = B(-3)$ so $B = -1$

C.
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$$\int \frac{3x^2+x+9}{x^3+9x}\,dx = \int\left(\frac{1}{x}+\frac{2x+1}{x^2+9}\right)dx = \int\left(\frac{1}{x}+\frac{2x}{x^2+9}+\frac{1}{x^2+9}\right)dx$$

$$= \ln|x| + \ln(x^2+9) + \frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$$

$$\frac{3x^2+x+9}{x(x^2+9)} = \frac{A}{x}+\frac{Bx+C}{x^2+9}$$

$$3x^2+x+9 = A(x^2+9)+(Bx+C)x$$

Let $x=0$, $9 = A(9) \Rightarrow A=1$

Equate coeff: $x^2: \ 3 = A+B \Rightarrow B=2$

$x: \ 1 = C$

D.

$$\int \frac{4x^2+5x+3}{x(x+1)^2}\,dx = \int\left(\frac{3}{x}+\frac{1}{x+1}-\frac{2}{(x+1)^2}\right)dx$$

$$= 3\ln|x| + \ln|x+1| + \frac{2}{x+1}+C$$

$$\frac{4x^2+5x+3}{x(x+1)^2} = \frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

$$4x^2+5x+3 = A(x+1)^2+Bx(x+1)+Cx$$

let $x=0$, $3=A$

Equate coeff: $x^2: \ 4 = A+B \Rightarrow B=1$

$x: \ 5 = 2A+B+C \Rightarrow C=-2$

E.

$$\int \frac{x^3-4x^2+10x}{x^2-4x+8}\,dx = \int\left(x+\frac{2x}{x^2-4x+8}\right)dx = \int\left(x+\frac{2x-4}{x^2-4x+8}+\frac{4}{x^2-4x+8}\right)dx$$

split so that we can do this

u-sub on the middle part

$u = x^2-4x+8$
$du = (2x-4)\,dx$

$$\begin{array}{r} x \\ x^2-4x+8 \overline{\big)\, x^3-4x^2+10x} \\ \underline{x^3-4x^2+8x} \\ 2x \end{array}$$

$$= \int\left(x+\frac{2x-4}{x^2-4x+8}+\frac{4}{(x-2)^2+2^2}\right)dx$$

$$= \frac{x^2}{2} + \ln|x^2-4x+8| + 2\arctan\left(\frac{x-2}{2}\right)+C$$

F. $$\int \frac{\sin x}{\cos x - \cos^2 x}\,dx = -\int \frac{du}{u-u^2} = \int \frac{du}{u^2-u} = \int \left(\frac{1}{u-1} - \frac{1}{u}\right) du = \ln|u-1| - \ln|u| + C$$

$$= \ln|\cos x - 1| - \ln|\cos x| + C$$

$u = \cos x$

$du = -\sin x\,dx$

$$\frac{1}{u^2-u} = \frac{A}{u-1} + \frac{B}{u}$$

$$1 = Au + B(u-1)$$

Let $u=0$, $1 = B(-1) \Rightarrow B=-1$

$u=1$, $1 = A \Rightarrow A=1$

1.E $$\int_{-\infty}^{0} xe^x\,dx = xe^x\Big|_{-\infty}^{0} - \int_{-\infty}^{0} e^x\,dx = \left(xe^x - e^x\right)\Big|_{-\infty}^{0} = \lim_{R\to-\infty}\left(xe^x - e^x\right)\Big|_{R}^{0}$$

$u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

$$= \lim_{R\to-\infty}\left(0 - 1 - Re^R + e^R\right)$$

$$= -1 - \lim_{R\to-\infty}\left(Re^R\right)$$

$(-\infty)(0)$ form which is indeterminate, so we use L'Hopital's Rule

$$= -1 - \lim_{R\to-\infty}\left(\frac{R}{e^{-R}}\right)$$

$\frac{(-\infty)}{\infty}$ form, so L'H applies

$$\overset{L'H}{=} -1 - \lim_{R\to-\infty}\left(\frac{1}{-e^{-R}}\right) = -1$$

6.A $$\int_{4}^{\infty} \frac{5}{x^2-x-6}\,dx = \int_{4}^{\infty}\left(\frac{1}{x-3} - \frac{1}{x+2}\right)dx = \lim_{R\to\infty}\left(\ln|x-3| - \ln|x+2|\right)\Big|_{4}^{R}$$

$$\frac{5}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$5 = A(x+2) + B(x-3)$$

Let $x=3$, $5 = 5A \Rightarrow A=1$

$x=-2$, $5 = -5B \Rightarrow B=-1$

$$= \lim_{R\to\infty}\left(\underbrace{\ln|R-3| - \ln|R+2|}_{\infty - \infty \text{ indeterminate form}} - \ln|4-3| + \ln|4+2|\right)$$

$$= \lim_{R\to\infty} \ln\left|\frac{R-3}{R+2}\right| + \ln 6$$

$$= \ln\left(\lim_{R\to\infty}\frac{R-3}{R+2}\right) + \ln 6 = \ln 6$$

Since $\ln(\cdot)$ is continuous, the limit & the function can switch.

1.J $$\int_0^\infty \frac{2}{3x+5}\,dx = \lim_{R\to\infty} \frac{2}{3}\ln|3x+5|\Big|_0^R = \lim_{R\to\infty} \frac{2}{3}\ln|3R+5| - \frac{2}{3}\ln 5 = \infty.$$

so the integral diverges.

6.B $$\int_0^\infty \frac{\arctan x}{x^2+1}\,dx = \lim_{R\to\infty} \frac{1}{2}\arctan^2 x\Big|_0^R = \lim_{R\to\infty} \frac{1}{2}\arctan^2 R - 0 = \frac{1}{2}\left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{8}$$

1.O $$\int_0^1 \ln x\,dx = x\ln x\Big|_0^1 - \int_0^1 dx = \lim_{R\to 0^+} (x\ln x - x)\Big|_R^1 = \lim_{R\to 0^+}\left(-1 - R\ln R + R\right)$$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$$= -1 - \lim_{R\to 0^+} (R\ln R) \longleftarrow 0\cdot(-\infty) \text{ indeterminate form}$$

$$= -1 - \lim_{R\to 0^+}\left(\frac{\ln R}{1/R}\right) \longleftarrow \frac{-\infty}{\infty} \text{ indeterminate form,}$$

L'H applies

$$\overset{L'H}{=} -1 - \lim_{R\to 0^+}\left(\frac{1/R}{-1/R^2}\right) = -1 - \lim_{R\to 0^+}(-R) = -1$$

6.C 2 $$\int_{-5}^{2} \frac{dx}{\sqrt[5]{x-2}} = \int_{-7}^{0} u^{-1/5}\,du = \lim_{R\to 0^-} \frac{5}{4}u^{4/5}\Big|_{-7}^R = \lim_{R\to 0^-}\left(\frac{5}{4}(R)^{4/5} - \frac{5}{4}(-7)^{4/5}\right)$$

Let $u = x-2$

$du = dx$

$x=-5 \mapsto u=-7$

$x\to 2 \mapsto u\to 0$

$$= -\frac{5}{4}(-7)^{4/5}$$