Math 172 | Exam 2 | Name: ____________

9 June 2016 | Reasonably Simplify | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{10}$ Evaluate the following improper integral, or show it diverges.

$$\int_0^\infty \frac{2}{x^2+6x+8}\,dx$$

Hint: $\dfrac{2}{x^2+6x+8} = \dfrac{1}{x+2} - \dfrac{1}{x+4}$

$$\int_0^\infty \frac{2}{x^2+6x+8}\,dx = \int_0^\infty \left(\frac{1}{x+2} - \frac{1}{x+4}\right)dx = \lim_{R\to\infty} \ln|x+2| - \ln|x+4| \Big|_0^R$$

$$= \lim_{R\to\infty} \ln\left|\frac{x+2}{x+4}\right| \Bigg|_0^R = \lim_{R\to\infty} \ln\left|\frac{R+2}{R+4}\right| - \ln\left|\frac{2}{4}\right|$$

$$= -\ln\frac{1}{2} = \ln 2$$

2. $\boxed{10}$ Use the Comparison Test to show the following integral converges.

$$\int_2^\infty \frac{\cos^2 x}{x(x^2+2)}\,dx$$

Since $0 \leq \dfrac{\cos^2 x}{x(x^2+2)} \leq \dfrac{1}{x^3}$ & $\displaystyle\int_2^\infty \frac{dx}{x^3}$ is a convergent p-integral ($p=3>1$), by the Comparison Test

$$\int_2^\infty \frac{\cos^2 x}{x(x^2+2)}\,dx \text{ also converges.}$$

3. Consider the parametrically defined curve

$$x(t) = \tan t, \quad y(t) = \sin^2 t.$$

(a) $\boxed{4}$ Find the slope of the curve, i.e. dy/dx, at $t = \pi/3$.

$x' = \sec^2 t$ $\qquad x'(\pi/3) = 4$

$y' = 2\sin t \cos t$ $\qquad y'(\pi/3) = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

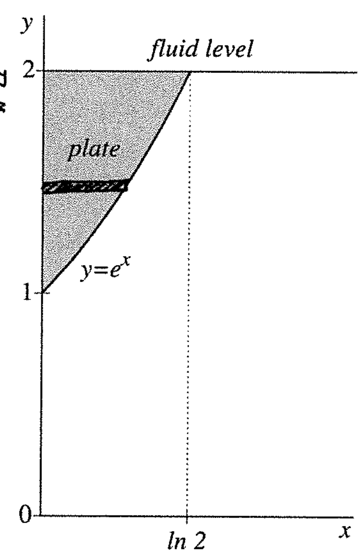
$y'/x' = \frac{dy}{dx} = \frac{\sqrt{3}}{8}$

(b) $\boxed{4}$ Find the speed of the curve, i.e. ds/dt, at $t = \pi/4$.

$x'\left(\frac{\pi}{4}\right) = 2$

$y'\left(\frac{\pi}{4}\right) = 1$ $\qquad \frac{ds}{dt} = \sqrt{5}$

4. $\boxed{12}$ Calculate the fluid force on one side of a plate in the shape of the region shown in the figure. The surface of the mystery fluid of density ρ is at $y = 2$.



$$\rho g \int_1^2 (2-y)\ln y \, dy$$

$u = \ln y \qquad dv = (2-y)\,dy$

$du = \frac{1}{y}\,dy \qquad v = 2y - \frac{y^2}{2}$

$$= \rho g\left[\left(2y - \frac{y^2}{2}\right)\ln y\Big|_1^2 - \int_1^2 \left(2 - \frac{y}{2}\right) dy\right]$$

$$= \rho g\left[2\ln 2 - \left(2y - \frac{y^2}{4}\right)\Big|_1^2\right]$$

$$= \rho g\left[2\ln 2 - \left[4 - 1 - 2 + \frac{1}{4}\right]\right] = \rho g\left[2\ln 2 - \frac{5}{4}\right]$$

5. $\boxed{3}$ Find parametric equations, $x(t)$ and $y(t)$, for the line segment from $(-2, 0)$ to $(1, -4)$. Include the domain of t.

$x = -2 + 3t$

$y = 0 - 4t$ for $0 \leq t \leq 1$

6. $\boxed{3}$ Find parametric equations, $x(t)$ and $y(t)$, for the circle with center $(4, -3)$ and radius 5. Include the domain of t.

$x = 4 + 5\cos t$

$y = -3 + 5\sin t$

7. $\boxed{10}$ Find the length of the parametric curve.

$$x = t^4, \quad y = \frac{t^7}{7} - 4t, \quad 0 \leq t \leq 1$$

$x' = 4t^3 \qquad y' = t^6 - 4$

$(x')^2 + (y')^2 = 16t^3 + t^{12} - 8t^6 + 16 = t^{12} + 8t^6 + 16 = (t^6 + 4)^2$

$ds = (t^6 + 4)\,dt$

$$S = \int_0^1 (t^6 + 4)\,dt = \frac{t^7}{7} + 4t \Big|_0^1 = \frac{1}{7} + 4 = \frac{29}{7}$$

8. $\boxed{10}$ Find the arc length of $y = \pi + x^2$ for $0 \le x \le 1/2$.

$$y' = 2x \qquad ds = \sqrt{1 + (2x)^2}\, dx$$

$$S = \int_0^{1/2} \sqrt{1+(2x)^2}\, dx = \frac{1}{2}\int_0^{\pi/4} \sec^3\theta\, d\theta = \frac{1}{4}\left(\tan\theta \sec\theta + \ln|\sec\theta + \tan\theta|\right)\Big|_0^{\pi/4}$$

Let $2x = \tan\theta$

$2dx = \sec^2\theta\, d\theta$

x		θ
0	$\longmapsto$	0
$\frac{1}{2}$	$\longmapsto$	$\frac{\pi}{4}$

$$= \frac{1}{4}\left(\sqrt{2} + \ln\left|\sqrt{2} + 1\right|\right)$$

9. $\boxed{10}$ Find the length of the polar curve.

$$r = \theta^2, \quad 0 \le \theta \le \pi$$

$$r' = 2\theta$$

$$ds = \sqrt{\theta^4 + 4\theta^2}\, d\theta = \theta\sqrt{\theta^2+4}\, d\theta$$

$$S = \int_0^{\pi} \theta\sqrt{\theta^2+4}\, d\theta = \frac{1}{2}\int_4^{\pi^2+4} u^{1/2}\, du = \frac{1}{3}(u)^{3/2}\Big|_4^{\pi^2+4}$$

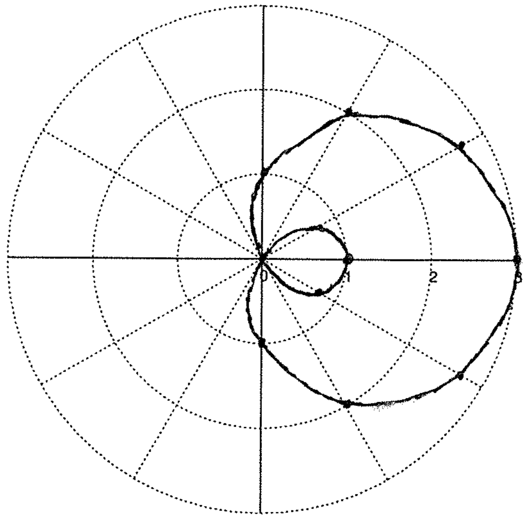
$u = \theta^2 + 4$

$du = 2\theta\, d\theta$

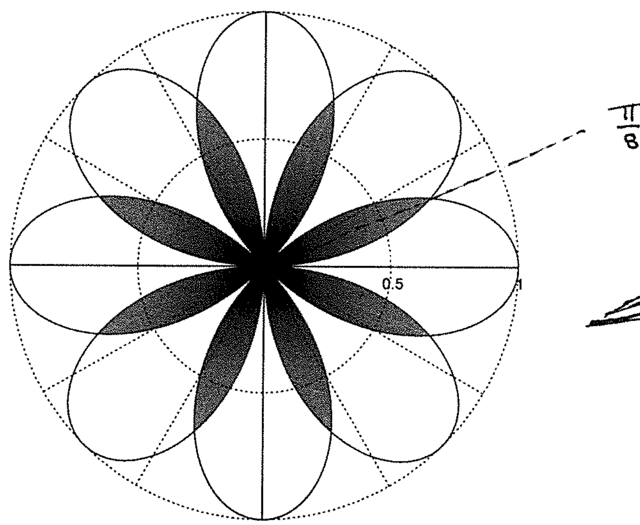
$$= \frac{1}{3}\left((\pi^2+4)^{3/2} - 4^{3/2}\right)$$

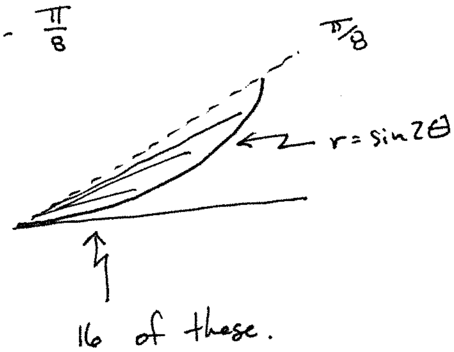
$$= \frac{1}{3}\left((\pi^2+4)^{3/2} - 8\right)$$

10. $\boxed{4}$ Carefully sketch the polar curve $r = 2\cos\theta - 1$.



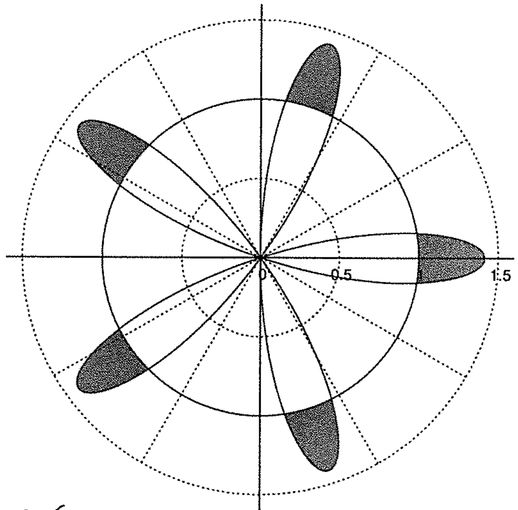
11. $\boxed{8}$ Find an integral representing the area inside both $r = \sin 2\theta$ and $r = \cos 2\theta$. The shaded region in the figure below. **Do not evaluate the integral.**

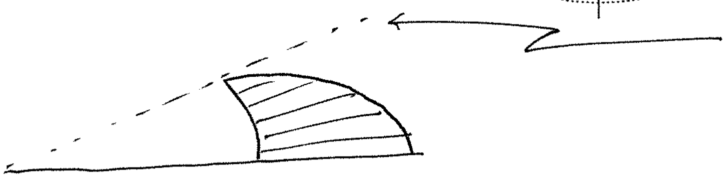




$$16 \cdot \frac{1}{2} \int_0^{\pi/8} \sin^2 2\theta \, d\theta$$

12. $\boxed{12}$ Find the area inside $r = \sqrt{2}\cos 5\theta$ and outside $r = 1$, the shaded region in the figure.





10 of these

intersect when

$1 = \sqrt{2}\cos 5\theta$

$\frac{1}{\sqrt{2}} = \cos 5\theta$

so $5\theta = \pi/4$

$\theta = \pi/20$

$$10 \cdot \frac{1}{2} \int_0^{\pi/20} \left[2\cos^2 5\theta - 1 \right] d\theta = 5 \int_0^{\pi/20} \cos 10\theta \, d\theta$$

$$= \frac{5}{10} \sin 10\theta \Big|_0^{\pi/20} = \frac{1}{2}$$