

1. 5 Find the sum of the series, or state the series diverges.

$$(a) \sum_{n=2}^{\infty} \frac{2}{5^n} = \frac{\frac{2}{25}}{1 - \frac{1}{5}} = \frac{\frac{2}{25}}{\frac{4}{5}} = \frac{2}{25} \cdot \frac{5}{4} = \frac{1}{10}$$

↑

$$= \sum_{n=2}^{\infty} 2 \left(\frac{1}{5}\right)^n = \frac{\text{first term}}{1 - \text{ratio}}$$

$$(b) \sum_{n=0}^{\infty} \frac{1 + (-2)^n}{2^{2n}} = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n + \sum_{n=0}^{\infty} \left(\frac{-2}{4}\right)^n$$

↑

$$2^{2n} = 4^n$$
$$= \frac{1}{1 - \frac{1}{4}} + \frac{1}{1 - \left(-\frac{1}{2}\right)}$$
$$= \frac{1}{\frac{3}{4}} + \frac{1}{\frac{3}{2}} = \frac{4}{3} + \frac{2}{3} = 2$$

Continued on the other side.

2. 3 Find the values of x for which the following series converges and find what it converges to.

$$\sum_{n=0}^{\infty} \frac{3x^n}{2^n} = \sum_{n=0}^{\infty} 3 \left(\frac{x}{2}\right)^n$$

$$= \frac{3}{1 - \frac{x}{2}} \quad \text{for } \left|\frac{x}{2}\right| < 1$$

$$= \frac{6}{2-x} \quad \text{for } |x| < 2$$

3. 2 How big is a Sierpiński Carpet? We discussed two different ways to answer this question yesterday, either is sufficient. Provide appropriate justification for your answer.

$$\text{Area} = 0 \quad \text{since} \quad 1 - \left(\frac{1}{9} + \frac{8}{9^2} + \frac{8^2}{9^3} + \dots\right) = 1 - \left(\frac{1/9}{1 - 8/9}\right) = 1 - 1 = 0$$

$$\dim_S S = \frac{\ln 8}{\ln 3}$$

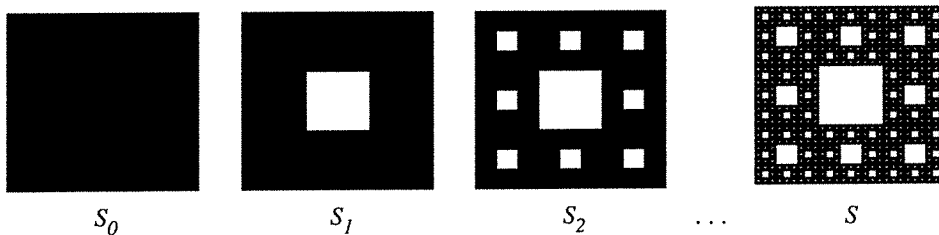


Figure 1: Sierpiński carpet