1. Use the geometric power series

\[
\frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots \quad \text{for } |x| < 1
\]

to find a power series representation for the following functions. Specify where each series converges.

(a) \[ g(x) = \frac{3x}{1 + x} = 3x \cdot \frac{1}{1 - (-x)} = 3x \sum_{n=0}^{\infty} (-x)^n \]

\[ = \sum_{n=0}^{\infty} 3(-1)^n x^{n+1} \]

\[
\sum_{n=0}^{\infty} 3(-1)^n x^{n+1} \quad \text{for } |x| < 1.
\]

(b) \[ f(x) = \frac{4}{2 + x} \quad \text{centered at } x = 2 \]

\[
= \frac{4}{2 + (x - 2) + 2} = \frac{4}{1 + (x - 2)} = \left| \frac{4}{1 - \left(\frac{x - 2}{-4}\right)} \right| 
\]

\[
= \sum_{n=0}^{\infty} \left(\frac{x - 2}{-4}\right)^n \quad \text{for } \left|\frac{x - 2}{-4}\right| < 1.
\]

\[ \text{i.e. } \left| x - 2 \right| < 4 \]
2. Next week we will show

\[
\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n + 1)!} \quad \text{for all } x \in \mathbb{R}.
\]

Use this series representation to integrate the following.

\[
\int \sin x^2 \, dx
\]

\[
\sin x^2 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}
\]

\[
\int \sin x^2 \, dx = A + \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}
\]