Math 172 | Quiz 16 | Name: ____________

16 June 2016 | Reasonably Simplify | Point Values in $\boxed{\text{boxes}}$.

1. Use the geometric power series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots \qquad \text{for } |x| < 1$$

to find a power series representation for the following functions. Specify where each series converges.

(a) $\boxed{3}$ $g(x) = \dfrac{3x}{1+x}$ $= 3x \cdot \dfrac{1}{1-(-x)} = 3x \sum_{n=0}^{\infty} (-x)^n$

$$= \sum_{n=0}^{\infty} 3(-1)^n x^{n+1}$$

for $|-x| < 1$ i.e. $|x| < 1$

(b) $\boxed{4}$ $f(x) = \dfrac{4}{2+x}$ centered at $x = 2$

$$= \frac{4}{2+x-2+2} = \frac{4}{4+(x-2)} = \frac{1}{1-\left(\frac{x-2}{-4}\right)}$$

$$= \sum_{n=0}^{\infty} \left(\frac{x-2}{-4}\right)^n \qquad \text{for } \left|\frac{x-2}{-4}\right| < 1$$

i.e. $|x-2| < 4$

**Continued on the other side.**

2. $\boxed{3}$ Next week we will show

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad \text{for all } x \in \mathbb{R}.$$

Use this series representation to integrate the following.

$$\int \sin x^2 \, dx$$

$$\sin x^2 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$$

$$\text{So} \quad \int \sin x^2 \, dx = A + \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$$