Math 172 | Quiz 17 | Name: ______________

20 June 2016 | Reasonably Simplify | Point Values in $\boxed{\text{boxes}}$.

1. Find the radius of convergence R for the following. Express your solution in the form $|x - c| < R$, where possible.

(a) $\boxed{3}$ $\displaystyle\sum_{n=1}^{\infty} \frac{4^n(x-1)^n}{n^2+1}$

Root Test

$$\sqrt[n]{|a_n|} = \frac{4|x-1|}{\sqrt[n]{n+1}} \xrightarrow[n\to\infty]{} 4|x-1| < 1 \quad \text{when} \quad |x-1| < \frac{1}{4}$$

(b) $\boxed{3}$ $\displaystyle\sum_{n=1}^{\infty} \frac{n^2(x-2)^n}{(n+1)!}$

Ratio Test

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)^2(x-2)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^2(x-2)^n}\right| = \left(\frac{n+1}{n}\right)^2 |x-2| \cdot \frac{1}{n+1} \xrightarrow[n\to\infty]{} 0$$

So the radius of convergence is ∞.

Continued on the other side.

2. $\boxed{4}$ Use the Comparison Test to show $\sum_{n=1}^{\infty} \frac{2n-1}{3n^4+n^2+5}$ converges.

Since $0 < \dfrac{2n-1}{3n^4+n^2+5} < \dfrac{2n}{3n^4} < \dfrac{1}{n^3}$

& $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges as a p-series with $p = 3 > 1$,

by the Comparison Test, $\sum_{n=1}^{\infty} \frac{2n-1}{3n^4+n^2+5}$ also converges.