Math 172 | Quiz 18 | Name: ____________
21 June 2016 | Provide appropriate justification | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Please circle **True** or **False**, as appropriate.

   (a) **T** / **F:** If $\sum a_n$ converges conditionally, then $\sum |a_n|$ converges.

   (b) **T** / **F:** If $\sum a_n$ converges conditionally, then $\sum a_n$ converges.

   (c) **T** / **F:** If $\sum a_n$ converges absolutely, then $\sum |a_n|$ converges.

   (d) **T** / **F:** If $\sum a_n$ converges absolutely, then $\sum a_n$ converges.

2. $\boxed{8}$ Find the **interval of convergence** for the following power series. Provide appropriate justification for each claim. Use limits where appropriate.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{2^n (n+1)}$$

Root Test

$$\sqrt[n]{|a_n|} = \sqrt[n]{\left|\frac{(-1)^n (x-3)^n}{2^n (n+1)}\right|} = \frac{|x-3|}{2} \cdot \frac{1}{\sqrt[n]{n+1}} \xrightarrow[n\to\infty]{} \frac{|x-3|}{2}$$

so $|x-3| < 2$

$x = 5$
$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{2^n (n+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$$ which converges by the A.S.T. since i) $\frac{1}{n+1} > 0$, ii) $\frac{1}{n+1} > \frac{1}{n+2}$, and iii) $\frac{1}{n+1} \xrightarrow[n\to\infty]{} 0$

$x = 1$
$$\sum_{n=0}^{\infty} \frac{(-1)^n (-2)^n}{2^n (n+1)} = \sum_{n=0}^{\infty} \frac{1}{n+1}$$ which diverges by comparison since $0 < \frac{1}{2n} < \frac{1}{n+1}$ & $\sum_{n=1}^{\infty} \frac{1}{2n}$ is a divergent p-series

Interval $(1, 5]$