Math 172	Quiz 19	Name: ______________
22 June 2016	You know the drill.	Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{1}$ Integrate $\int \sin x^2\, dx$.

$$\sin x^2 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!} \quad \text{so} \quad \int \sin x^2\, dx = A + \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$$

2. $\boxed{1}$ Find the first 3 nonzero term of the Taylor series of $\sin 3x$ centered at $c = \pi/3$.

n	$f^{(n)}$	$f^{(n)}(\frac{\pi}{3})$
0	$\sin 3x$	0
1	$3\cos 3x$	-3
2	$-3^2 \sin 3x$	0
3	$-3^3 \cos 3x$	$+27$
⋮	⋮	0
		-243

$$\sin 3x = -3(x - \tfrac{\pi}{3}) + \frac{27}{3!}(x - \tfrac{\pi}{3})^3 - \frac{243}{5!}(x - \tfrac{\pi}{3})^5 \dots$$

3. $\boxed{3}$ Find the interval of convergence of the following power series. Little to no work is expected.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n x^n}{n^2+1}$ $\qquad [-\frac{1}{4}, \frac{1}{4}]$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^n}{(-3)^n(2n-1)}$ $\qquad (-4, 2]$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-5)^n}{(n+3)!}$ $\qquad (-\infty, \infty)$

Continued on the other side.

4. $\boxed{2}$ Find series that satisfy the given requirements.

(a) A series that converges but the Root Test is inconclusive.

$$\sum \frac{1}{n^2}$$

(b) A series that diverges but the Root Test is inconclusive.

$$\sum \frac{1}{n}$$

(c) A series that has sum 1/2.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (\pi/3)^{2n}}{(2n)!} = \cos(\pi/3)$$

(d) A series that is conditionally convergent.

$$\sum \frac{(-1)^n}{n}$$

5. $\boxed{1}$ For the following series, specify what series you would compare each to and based on your comparison, decide if it converges or diverges. No formal justification is needed.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{n+1}{\sqrt{n^3+4n}}$ compare to $\displaystyle\sum \frac{n}{n^{3/2}} = \sum \frac{1}{n^{1/2}}$ so it CONVERGES / DIVERGES

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{1+n}}{n^2+3n+1}$ compare to $\displaystyle\sum \frac{\sqrt{n}}{n^2} = \sum \frac{1}{n^{3/2}}$ so it CONVERGES / DIVERGES

6. $\boxed{1}$ Which of the following inequalities is useful in order to show $\displaystyle\sum \frac{2}{n+3}$ diverges?

(a) $0 < \dfrac{2}{n+3} < \dfrac{2}{n}$ (b) $0 < \dfrac{1}{n} = \dfrac{2}{n+n} < \dfrac{2}{n+3}$

(c) Neither

(d) Both

7. $\boxed{1}$ Use series to compute the following limit.

$$\lim_{x\to 0} \frac{x - \sin x}{\arctan x - x} = \lim_{x\to 0} \frac{x - \left(x - \frac{x^3}{3!} + \cdots\right)}{\left(x - \frac{x^3}{3} + \cdots\right)}$$

$$= \lim_{x\to 0} \frac{\frac{x^3}{3!} + \cdots}{-\frac{x^3}{3} + \cdots} = \lim_{x\to 0} \frac{1/3! + \cdots}{-1/3 + \cdots} = \frac{3}{3!} = \frac{1}{2}$$