Math 172
23 May 2016

Quiz 4
Show Appropriate Work

Name: _____
Point Values in boxes.

1. [3] Integrate.

$$\int 2x \sin 3x \, dx$$

$u = 2x$ $\quad dv = \sin 3x \, dx$

$du = 2\,dx$ $\quad v = -\frac{1}{3}\cos 3x$

$$= -\frac{2x}{3}\cos 3x + \int \frac{2}{3}\cos 3x \, dx$$

$$= -\frac{2x}{3}\cos 3x + \frac{2}{9}\sin 3x + C$$

2. [3] In class on Thursday we discussed that for integrals involving $\ln x$ it is usually appropriate to use Integration by Parts with $u = \ln x$. Do so below to integrate the following.

$$\int x^4 \ln x \, dx$$

$u = \ln x$ $\quad dv = x^4 \, dx$

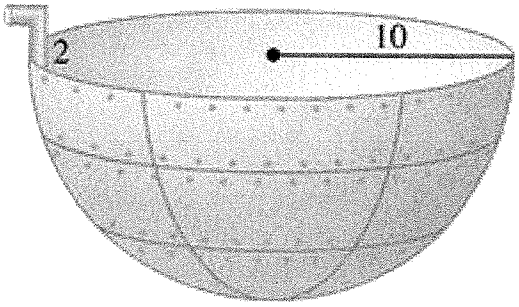
$du = \frac{1}{x}\,dx$ $\quad v = \frac{x^5}{5}$

$$= \frac{x^5 \ln x}{5} - \int \frac{x^4}{5}\,dx$$

$$= \frac{x^5 \ln x}{5} - \frac{x^5}{25} + C$$

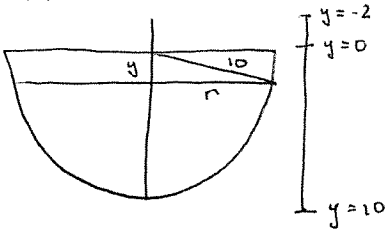
Continued on the other side.

3. [4] Consider the hemispherical tank with a spout in the figure below; the tank is filled with coffee of density ρ. Distances are in meters, the density of water is ρ, and acceleration due to gravity is g.



(a) Choose an appropriate coordinate system.

(b) Find the volume of a 'slice' of coffee.



$$V_i = \pi (r)^2 \Delta y = \pi (10^2 - y^2) \Delta y$$

(c) Find the force on a 'slice' of coffee.

$$\pi (10^2 - y^2) \rho g \Delta y$$

(d) Find the distance the 'slice' moves.

$$(y - (-2)) = y + 2$$

(e) Express, as an integral, the work (in joules) required to pump all of the coffee from the tank via the spout. **You do not need to evaluate the integral.**

$$\int_0^{10} \pi \rho g (100 - y^2)(y + 2)\, dy$$