1. \[ \int 2x \sin(3x) \, dx \]

\[ u = 2x \quad \text{and} \quad dv = \sin(3x) \, dx \]
\[ du = 2 \, dx \quad \text{and} \quad v = -\frac{1}{3} \cos(3x) \]

\[ = -\frac{2x}{3} \cos(3x) + \int \frac{2}{3} \cos(3x) \, dx \]

\[ = -\frac{2x}{3} \cos(3x) + \frac{2}{9} \sin(3x) + C \]

2. In class on Thursday we discussed that for integrals involving \( \ln x \) it is usually appropriate to use Integration by Parts with \( u = \ln x \). Do so below to integrate the following.

\[ \int x^4 \ln x \, dx \]

\[ u = \ln x \quad \text{and} \quad dv = x^4 \, dx \]
\[ du = \frac{1}{x} \, dx \quad \text{and} \quad v = \frac{x^5}{5} \]

\[ = \frac{x^5 \ln x}{5} - \int \frac{x^4}{5} \\ dx \]

\[ = \frac{x^5 \ln x}{5} - \frac{x^5}{25} + C \]

Continued on the other side.
3. Consider the hemispherical tank with a spout in the figure below; the tank is filled with coffee of density \( \rho \). Distances are in meters, the density of water is \( \rho \), and acceleration due to gravity is \( g \).

(a) Choose an appropriate coordinate system.

(b) Find the volume of a 'slice' of coffee.

\[
V_i = \pi \left( r \right)^2 \Delta y = \pi \left( 10^2 - y^2 \right) \Delta y
\]

(c) Find the force on a 'slice' of coffee.

\[
\pi \left( 10^2 - y^2 \right) g \Delta y
\]

(d) Find the distance the 'slice' moves.

\[
\left( y - (-2) \right) = y + 2
\]

(e) Express, as an integral, the work (in joules) required to pump all of the coffee from the tank via the spout. **You do not need to evaluate the integral.**

\[
\int_{0}^{10} \pi g \rho \left( 100 - y^2 \right) \left( y + 2 \right) dy
\]