1. \[ \int \cos^3 3x \sin^4 3x \, dx \]

\[ = \int \cos^2 3x \, \sin^4 3x \, \cos 3x \, dx \]

\[ = \int \left( 1 - \sin^2 3x \right) \sin^4 3x \, \cos 3x \, dx \]

\( u = \sin 3x \)

\( du = 3 \cos 3x \, dx \)

\[ = \frac{1}{3} \int \left( 1 - u^2 \right) u^4 \, du = \frac{1}{3} \left[ \frac{u^5}{5} - \frac{u^7}{7} \right] + C \]

\[ = \frac{\sin^5 3x}{15} - \frac{\sin^7 3x}{21} + C \]

2. In class on Monday we discussed that for integrals involving \( \sec^n \theta \tan^m \theta \) we can use the substitution \( u = \sec \theta \) or \( u = \tan \theta \) for many of them.

(a) If \( u = \sec \theta \) what is \( du \)?

\[ du = \sec \theta \tan \theta \, d\theta \]

(b) If \( u = \tan \theta \) what is \( du \)?

\[ du = \sec^2 \theta \, d\theta \]

(c) Using the appropriate substitution from above, integrate the following.

\[ \int \sec^4 \theta \tan^4 \theta \, d\theta \]

\[ = \int \sec^2 \theta \tan^4 \theta \, d\theta \]

\[ = \int \left( 1 + \tan^2 \theta \right) \tan^4 \theta \, d\theta \]

\( u = \tan \theta \)

\( du = \sec^2 \theta \, d\theta \)

\[ = \frac{1}{5} \left[ \tan^5 \theta \right] + \frac{1}{7} \left[ \tan^7 \theta \right] + C \]

\[ = \frac{1}{5} \frac{\tan^5 \theta}{\tan^2 \theta} + \frac{1}{7} \frac{\tan^7 \theta}{\tan^2 \theta} + C \]

Continued on the other side.
3. For each of the integrals below, choose an appropriate trigonometric substitution, compute the corresponding \(dx\) term, and draw the associated triangle - please label all three sides. **Do not integrate.**

\[
\int \sqrt{9 + x^2} \, dx
\]

(a) The appropriate trigonometric substitution.

\[\tan \theta = \frac{x}{3}\]

(b) The corresponding \(dx\) term.

\[dx = 3 \sec^2 \theta \, d\theta\]

(c) The associated triangle.

\[
\int \sqrt{4x^2 - 1} \, dx
\]

(a) The appropriate trigonometric substitution.

\[\sec \theta = 2x\]

(b) The corresponding \(dx\) term.

\[2dx = \sec \theta \tan \theta \, d\theta\]

(c) The associated triangle.