Math 172 | Quiz 5 | Name: ________________

23 May 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{3}$ Integrate.

$$\int \cos^3 3x \sin^4 3x \, dx$$

$$= \int \cos^2 3x \sin^4 3x \cos 3x \, dx = \int (1 - \sin^2 3x) \sin^4 3x \cos 3x \, dx$$

$u = \sin 3x$

$du = 3 \cos 3x \, dx$

$$= \frac{1}{3} \int (1 - u^2) u^4 \, du = \frac{1}{3} \left[\frac{u^5}{5} - \frac{u^7}{7} \right] + C$$

$$= \frac{\sin^5 3x}{15} - \frac{\sin^7 3x}{21} + C$$

2. $\boxed{3}$ In class on Monday we discussed that for integrals involving $\sec^n \theta \tan^m \theta$ we can use the substitution $u = \sec \theta$ or $u = tan\theta$ for many of them.

(a) If $u = \sec \theta$ what is du?

$du = \sec\theta \tan\theta \, d\theta$

(b) If $u = \tan \theta$ what is du?

$du = \sec^2\theta \, d\theta$

(c) Using the appropriate substitution from above, integrate the following.

$$\int \sec^4 \theta \tan^4 \theta \, d\theta$$

$$= \int \sec^2\theta \tan^4\theta \sec^2\theta \, d\theta = \int (1 + \tan^2\theta) \tan^4\theta \sec^2\theta \, d\theta$$

$u = \tan\theta \qquad du = \sec^2\theta \, d\theta$

$$= \int (1 + u^2) u^4 \, du = \frac{u^5}{5} + \frac{u^7}{7} + C$$

$$\cancel{= \frac{1}{5} \sec^5} = \frac{1}{5} \tan^5\theta + \frac{1}{7} \tan^7\theta + C$$

Continued on the other side.

3. $\boxed{4}$ For each of the integrals below, choose an appropriate trigonometric substitution, compute the corresponding 'dx' term, and draw the associated triangle - please label all three sides. **Do not integrate.**

$$\int \sqrt{9 + x^2}\, dx$$

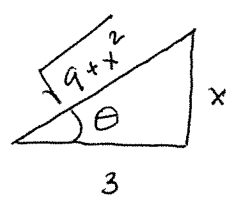
(a) The appropriate trigonometric substitution.

Let $x = 3 \tan \theta$

(b) The corresponding 'dx' term.

$dx = 3 \sec^2 \theta \, d\theta$

(c) The associated triangle.



$$\int \sqrt{4x^2 - 1}\, dx$$

(a) The appropriate trigonometric substitution.

Let $2x = \sec \theta$

(b) The corresponding 'dx' term.

$2dx = \sec \theta \tan \theta \, d\theta$

(c) The associated triangle.