

1. Integrate.

$$\int \sqrt{4-x^2} dx$$

$$\text{Let } x = 2 \sin \theta$$

$$dx = 2 \cos \theta d\theta$$

$$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta}$$

$$= \sqrt{4\cos^2\theta}$$

$$= 2 \cos \theta$$

so

$$\int \sqrt{4-x^2} dx = \int 4\cos^2\theta d\theta = 2 \int (1 + \cos 2\theta) d\theta = 2\left(\theta + \frac{1}{2} \sin 2\theta\right) + C$$

$$= 2\left(\theta + \sin\theta \cos\theta\right) + C$$

[You may find the identity $\sin 2\theta = 2 \sin \theta \cos \theta$ useful.]

$$= 2\left(\arcsin\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{4}\right) + C$$

