Math 172 | Quiz 8 | Name: ______________
1 June 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{5}$ Evaluate the following improper integral.

$$\int_1^\infty \frac{dx}{x^2+5x+6}$$

$$\frac{1}{(x+3)(x+2)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$1 = A(x+3) + B(x+2)$$

$x=-3$, $B=-1$

$x=-2$, $A=1$

$$= \int_1^\infty \left(\frac{1}{x+2} - \frac{1}{x+3} \right) dx$$

$$= \lim_{R\to\infty} \Big[\ln|x+2| - \ln|x+3| \Big] \Big|_1^R$$

$$= \lim_{R\to\infty} \ln\left|\frac{x+2}{x+3}\right| \Bigg|_1^R$$

$$= \lim_{R\to\infty} \ln\left|\frac{R+2}{R+3}\right| - \ln\left|\frac{3}{4}\right|$$

$$= -\ln\left|\frac{3}{4}\right| = \ln\left|\frac{4}{3}\right|$$

2. $\boxed{5}$ Use the Comparison Test to show the following integral converges.

$$\int_1^\infty \frac{2}{x^3+3x+4}\,dx$$

Since $0 \le \frac{2}{x^3+3x+4} \le \frac{2}{x^3}$ and $\int_1^\infty \frac{2}{x^3}\,dx$ is a convergent p-integral $(p=3>1)$, by comparison

$\int_1^\infty \frac{2}{x^3+3x+4}\,dx$ also converges.