Math 172 | Quiz 9 | Name: ______________

2 June 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ p-Integrals.

   (a) $\displaystyle\int_1^\infty \frac{dx}{x^p}$ converges for p >1 and diverges for p ≤ 1.

   (b) $\displaystyle\int_0^1 \frac{dx}{x^p}$ converges for p <1 and diverges for p ≥ 1.

2. $\boxed{4}$ Use the Comparison Test to show the following integral converges.

$$\int_0^1 \frac{2}{\sqrt{x}(x^3+2)}\,dx$$

Since $0 < \dfrac{2}{\sqrt{x}(x^3+2)} < \dfrac{2}{2\sqrt{x}} = \dfrac{1}{\sqrt{x}}$ for $x > 0$

& $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}}$ is a convergent p-integral $(p = \tfrac{1}{2} < 1)$,

by comparison $\displaystyle\int_0^1 \frac{2}{\sqrt{x}(x^3+2)}\,dx$ also converges.

3. $\boxed{4}$ Find the surface area generated by rotating the graph of $y = \sqrt{9 - x^2}$ for $-1 \leq x \leq 1$ about the x-axis. Note: this is part of a sphere.

$y = \sqrt{9-x^2}$

$y' = \frac{1}{2}(9-x^2)^{-1/2}(-2x)$

$= \dfrac{-x}{\sqrt{9-x^2}}$

$1 + (y')^2 = 1 + \dfrac{x^2}{9-x^2}$

$= \dfrac{9-x^2+x^2}{9-x^2}$

$= \dfrac{9}{9-x^2}$

so $ds = \sqrt{\dfrac{9}{9-x^2}}\,dx = \dfrac{3}{\sqrt{9-x^2}}\,dx$

$S = \displaystyle\int_{-1}^{1} 2\pi\sqrt{9-x^2}\cdot\frac{3}{\sqrt{9-x^2}}\,dx = 6\pi\int_{-1}^{1} dx = 12\pi$