

My hope is that you can finish this in class working in small groups. If you do not, please finish it and hand it in on Thursday. You may use the Math Learning Center.

Integrate.

$$\begin{aligned}
 1. \boxed{5} \int \frac{2+x}{\sqrt{9-x^2}} dx &= \int \frac{2}{\sqrt{9-x^2}} dx - \frac{1}{2} \int \frac{-2x}{\sqrt{9-x^2}} dx = 2 \int \frac{\frac{1}{3} dx}{\sqrt{1-(\frac{x}{3})^2}} - \frac{1}{2} \int u^{-\frac{1}{2}} du \\
 u &= 9-x^2 & v &= \frac{x}{3} \\
 du &= -2x dx & dv &= \frac{1}{3} dx \\
 &= 2 \int \frac{du}{\sqrt{1-u^2}} - u^{\frac{1}{2}} = 2 \arcsin v - \sqrt{9-x^2} + C \\
 &= 2 \arcsin \left( \frac{x}{3} \right) - \sqrt{9-x^2} + C
 \end{aligned}$$

$$2. \boxed{5} \int \frac{dx}{\sqrt{1+\sqrt{x}}} = \int \frac{2(u-1)}{\sqrt{u}} du = 2 \int (u^{\frac{1}{2}} - u^{-\frac{1}{2}}) du = 2 \left( \frac{2}{3} (1+\sqrt{x})^{\frac{3}{2}} - 2 (1+\sqrt{x})^{\frac{1}{2}} \right) + C$$

$$u = 1 + \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

so

$2\sqrt{x} du = dx$   $\leftarrow$  Note: Generally it is considered poor form to mix  $x$  &  $u$ . On occasion it is necessary though.  
i.e.

$$2(u-1)du = dx$$

In this case we could have written,

$$\text{let } u = 1 + \sqrt{x}$$

$$\text{so } (u-1)^2 = x$$

$$\text{so } 2(u-1)du = dx$$

in order to avoid it.

3. [4] Given that  $\int \frac{dx}{1+x^2} = \arctan x + C$ , show that for  $a > 0$ ,  $\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$ . This general form will come up regularly this semester.

$$\int \frac{du}{a^2+u^2} = \frac{1}{a^2} \int \frac{du}{1+\left(\frac{u}{a}\right)^2} = \frac{1}{a} \int \frac{dt}{1+t^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$$

$$\text{Let } t = \frac{u}{a}$$

$$dt = \frac{1}{a} du$$

4. [1] Use the integration formula you derived above to integrate the following.

$$\int \frac{e^x}{5+e^{2x}} dx \quad u = e^x \\ du = e^x dx$$

$$= \frac{1}{\sqrt{5}} \arctan\left(\frac{e^x}{\sqrt{5}}\right) + C$$

5. [5] We have seen only a small sample of possible substitutions so far. Integrate the following by starting with the substitution  $u^2 = x^2 - 1$ . [HINT:  $u^2 = u^2 + 0$ , write 0 in a useful way.]

$$\int \frac{\sqrt{x^2-1}}{x} dx$$

$$u^2 = x^2 - 1 \\ 2u du = 2x dx$$

$$\begin{aligned} &= \int \frac{\sqrt{x^2-1} \cdot x}{x^2} dx \\ &= \int \frac{u \cdot u du}{u^2+1} \end{aligned}$$

$$= \int \left( \frac{u^2+1}{u^2+1} - \frac{1}{u^2+1} \right) du$$

$$= u - \arctan u + C$$

$$= \sqrt{x^2-1} - \arctan \sqrt{x^2-1} + C$$