

1. [3] Consider the sequence $a_n = \arctan\left(\frac{n^2 + 3}{2n + n^2}\right)$.

(a) Evaluate $\lim_{n \rightarrow \infty} a_n$.

$$\lim_{n \rightarrow \infty} \arctan\left(\frac{n^2 + 3}{n^2 + 2n}\right) = \arctan(1) = \frac{\pi}{4}$$

(b) Does the sequence $\{a_n\}$ converge? yes, to $\frac{\pi}{4}$

(c) Does the series $\sum a_n$ converge? Why or why not? no, by the Test for Divergence.

2. [2] Find the sum.

$$4 - \frac{8}{5} + \frac{16}{25} - \frac{32}{125} + \frac{64}{625} - \dots = \frac{4}{1 - (-\frac{4}{5})} = \frac{4}{\frac{1}{5}} = \frac{20}{7}$$

3. [2] For each statement, determine if the use of the Comparison Test is Valid or Invalid.

(a) V / I : Since $0 < \frac{1}{n} < \frac{1}{n-1}$ and $\sum \frac{1}{n}$ diverges, by comparison $\sum \frac{1}{n-1}$ also diverges.

(b) V / I : Since $0 < \frac{1}{n+3} < \frac{1}{n}$ and $\sum \frac{1}{n}$ diverges, by comparison $\sum \frac{1}{n+3}$ also diverges.

4. [3] Find an example of a sequence with the following properties. If no such sequence exists explain why not.

(a) A bounded divergent sequence.

$$\{1, -1, 1, -1, 1, -1, \dots\}$$

(b) An increasing convergent sequence.

$$\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots\}$$

(c) A bounded decreasing divergent sequence.

Not possible, bounded monotone sequences converge.

5. [10] Use an appropriate series test to show each of the following series converge or diverge.

(a)

$$\text{Since } \frac{1}{n\sqrt{\ln n}} > \frac{1}{(n+1)\sqrt{\ln(n+1)}} \text{ if } \frac{1}{n\sqrt{\ln n}} > 0 \text{ for all } n \geq 2,$$

the C.C.T applies.

$$\sum_{k=1}^{\infty} 2^k \frac{1}{2^k \sqrt{\ln 2^k}} = \sum_{k=1}^{\infty} \frac{1}{\sqrt{k} \cdot \sqrt{\ln 2}} \text{ which is a divergent p-series (} p = \frac{1}{2} \leq 1\text{),}$$

$$\text{so by Cauchy, } \sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}} \text{ also diverges}$$

(b)

$$\sum_{n=2}^{\infty} \frac{n^2 + n}{n^4 - 7}$$

Since $\frac{n^2 + n}{n^4 - 7}, \frac{1}{n^2} > 0$, we can apply the L.C.T.

Since $\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - 7} / \frac{1}{n^2} = 1$ which is positive & finite,

both series have the same behavior. We know $\sum \frac{1}{n^2}$

is a convergent p-series, so $\sum_{n=2}^{\infty} \frac{n^2 + n}{n^4 - 7}$ also converges.