

$$I.A. \int_{-\infty}^{-4} \frac{5}{x^2 - x - 6} dx = \int_{-\infty}^{-4} \left[\frac{1}{x-3} - \frac{1}{x+2} \right] dx = \ln|x-3| - \ln|x+2| \Big|_{-\infty}^{-4}$$

$$\frac{5}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$5 = A(x+2) + B(x-3)$$

$$x=3 \Rightarrow A=1$$

$$x=-2 \Rightarrow B=-1$$

$$= \lim_{R \rightarrow \infty} \ln \left| \frac{x-3}{x+2} \right| \Big|_R^{-4}$$

$$= \lim_{R \rightarrow -\infty} \ln \left| \frac{-7}{-2} \right| - \ln \left| \frac{R-3}{R+2} \right|$$

$$= \ln \left(\frac{7}{2} \right) - \ln(1) = \ln \left(\frac{7}{2} \right)$$

$$B. \int_0^\infty \frac{1}{\sqrt{x}(x+1)} dx = 2 \int_0^\infty \frac{du}{1+u^2} = 2 \arctan(u) \Big|_0^\infty = \pi$$

$$\text{let } u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$\begin{matrix} 0 & \mapsto & 0 \\ x \rightarrow \infty & \mapsto & u \rightarrow \infty \end{matrix}$$

$$C. \int_{-5}^2 \frac{dx}{\sqrt[5]{x-2}} = \int_{-7}^0 \frac{du}{u^{1/5}} = \frac{5}{4} u^{4/5} \Big|_{-7}^0 = -\frac{5}{4} (-7)^{4/5}$$

$$\text{let } u = x-2$$

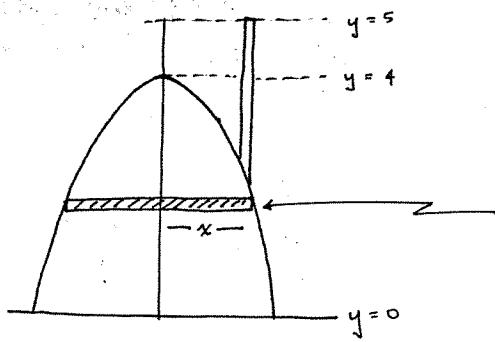
$$du = dx$$

$$\begin{matrix} 2 & \mapsto & 0 \\ -5 & \mapsto & -7 \end{matrix}$$

$$2. A. e^{t^2} = \sum_{n=0}^{\infty} \frac{t^{2n}}{n!} \quad \text{so} \quad \int_0^x e^{t^2} dt = A + \sum_{n=0}^{\infty} \frac{t^{2n+1}}{(2n+1)n!} \Big|_0^x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)n!}$$

$$B. \frac{\sin x}{x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} \quad \text{so} \quad \int \frac{\sin x}{x} dx = A + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$$

$$C. \sin(x^4) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{8n+4}}{(2n+1)!} \quad \text{so} \quad \int \sin(x^4) dx = A + \sum_{n=0}^{\infty} \frac{(-1)^n x^{8n+5}}{(8n+5)(2n+1)!}$$



$$\Delta W = \text{force} \pi (\text{radius})^2 \Delta y \quad (\text{distance}) \\ = \text{force} \pi x^2 (5-y) \Delta y \quad \text{but } x^2 = 4-y, \text{ so}$$

$$\text{Work} = \text{force} \pi \int_0^4 (4-y)(5-y) dy = \text{force} \pi \int_0^4 [y^2 - 9y + 20] dy = \text{force} \pi \left[\frac{y^3}{3} - \frac{9y^2}{2} + 20y \right] \Big|_0^4 \\ = \text{force} \pi \left[\frac{64}{3} - \cancel{\frac{72}{2}} + 80 \right] = \text{force} \pi \left[\frac{64}{3} + 8 \right] = \frac{88 \text{ force} \pi}{3}$$

4. A.

$$\sum_{n=3}^{\infty} \frac{2^{n+2}}{3^{2n}} = \frac{\frac{2^5}{3^6}}{1 - \frac{2}{9}} = \frac{32}{81 \cdot 9} \cdot \frac{9}{7} = \frac{32}{567}$$

\uparrow $\frac{\text{first term}}{1 - \text{ratio}}$

B.

$$\sum_{n=0}^{10} \frac{2^{2n}}{3^{n+1}} \text{ diverges since } r = \frac{4}{3} > 1$$

C.

$$\frac{1}{9} - \frac{8}{9^2} + \frac{8^2}{9^3} - \dots = \frac{\frac{1}{9}}{1 - (-\frac{8}{9})} = \frac{1}{9} \cdot \frac{9}{17} = \frac{1}{17}$$

5. $R = 5$

6. $\sum_{n=0}^{\infty} \left(\frac{x+9}{7} \right)^n$ is one such series.

7. $f(x) = \frac{4}{2+(x-2)+2} = \frac{4}{4+(x-2)} = \frac{1}{1 - \left(\frac{x-2}{4}\right)} = \sum_{n=0}^{\infty} \left(\frac{x-2}{4} \right)^n \text{ for } \left| \frac{x-2}{4} \right| < 1, \text{ i.e. } |x-2| < 4$

8. $x e^{x^3} + 2 \cos x^2 = x \left(1 + x^3 + \frac{x^6}{2} + \frac{x^9}{3!} + \dots \right) + 2 \left(1 - \frac{x^4}{2} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \dots \right)$

$$= 2 + x + \frac{x^2}{2} + \frac{2x^8}{4!} + \dots \Rightarrow f^{(16)}(0) = 0, \quad f^{(7)}(0) = \frac{7!}{2}$$

9. $x^2 \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+1)!} = x^3 - \dots + \frac{x^{23}}{21!} - \dots \text{ so } f^{(23)}(0) = \frac{23!}{21!} = 506 \quad \& \quad f^{(24)}(0) = 0$

10. $x^k e^x = \sum_{n=0}^{\infty} \frac{x^{n+k}}{n!} = \dots + \frac{n^j}{n!} + \dots \text{ where } j = n+k \text{ so } f^{(j)}(0) = \frac{j!}{n!} = \frac{j!}{(j-k)!}$