

Math 182
9 Feb 2017

Exam 1
Show Appropriate Work

Name: _____
Point Values in .

1. Integrate.

$$(a) \boxed{8} \int \tan^6 \theta \sec^4 \theta d\theta = \int \tan^6 \theta (\tan^2 \theta + 1) \sec^2 \theta d\theta = \int (u^6 + u^4) du$$

$$\begin{aligned} u &= \tan \theta \\ du &= \sec^2 \theta d\theta \end{aligned}$$

$$= \frac{u^7}{7} + \frac{u^9}{9} + C$$

$$= \frac{1}{7} \tan^7 \theta + \frac{1}{9} \tan^9 \theta + C$$

$$(b) \boxed{8} \int_0^{\pi^2} \cos \sqrt{x} dx = \int_0^{\pi} 2u \cos u du = 2u \sin u \Big|_0^{\pi} - \int_0^{\pi} 2 \sin u du = 2 \cos u \Big|_0^{\pi} = -4$$

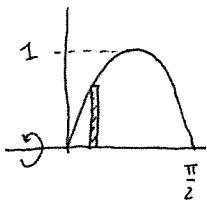
Let $u = \sqrt{x}$ $t = 2u$ $ds = \cos u du$
 so $u^2 = x$ $dt = 2 du$ $s = \sin u$
 $2u du = dx$

$x=0 \rightarrow u=0$
 $x=\pi^2 \rightarrow u=\pi$

$$(c) \boxed{8} \int \frac{x^5 + x^2}{5 + x^6} dx = \int \frac{x^5}{5 + x^6} dx + \int \frac{x^2}{5 + (x^3)^2} dx = \frac{1}{6} \ln(5 + x^6) + \frac{1}{3\sqrt{5}} \arctan\left(\frac{x^3}{\sqrt{5}}\right) + C$$

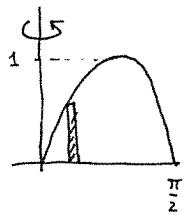
2. Consider the region bounded by the graph of $y = \sin 2x$ and the x -axis for $x \in [0, \pi/2]$.

(a) [8] The region is rotated about the x -axis, find the volume.



$$\begin{aligned} V &= \pi \int_0^{\frac{\pi}{2}} \sin^2 2x \, dx = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} (1 - \cos 4x) \, dx = \frac{\pi}{2} \left(x - \frac{1}{4} \sin 4x \right) \Big|_0^{\frac{\pi}{2}} \\ &= \frac{\pi}{2} \left(\frac{\pi}{2} \right) = \frac{\pi^2}{4} \end{aligned}$$

(b) [8] The region is rotated about the y -axis, find the volume.



$$\begin{aligned} V &= 2\pi \int_0^{\frac{\pi}{2}} x \sin 2x \, dx \\ u &= x & dv &= \sin 2x \, dx \\ du &= dx & v &= -\frac{1}{2} \cos 2x \\ &= 2\pi \left[-\frac{1}{2} x \cos 2x \Big|_0^{\frac{\pi}{2}} + \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos 2x \, dx \right] \\ &= 2\pi \left[-\frac{1}{2} \cdot \frac{\pi}{2} \cdot (-1) + \frac{1}{4} \sin 2x \Big|_0^{\frac{\pi}{2}} \right] \\ &= \frac{\pi^2}{4} \end{aligned}$$

3. [10] Soy sauce, canola oil, maple syrup, lemon juice, garlic powder, and cayenne pepper is mixed together for a tofu marinade. The mixture fills a spherical tank of radius 2 m. As it starts to settle, the density of the mixture is given by $\rho(y) = \kappa(1 + \sqrt{y})$ kg/m³ where y is the depth of the mixture in the tank, i.e. the density is κ kg/m³ at the top of the tank and $\kappa(1 + \sqrt{2})$ kg/m³ at the bottom. If the tank is drained from an opening in the top, express the work required as an integral. Use g m/s² for the gravitational constant. Unless you are bored, do NOT evaluate the integral.

$$x^2 + (y-z)^2 = 4$$

$$\text{so } x^2 = 4 - (y-z)^2$$

$$= 4 - y^2 + 2yz - z^2$$

$$= 4y - y^2$$

$$V_i = g \cdot K (1 + \sqrt{y}) \pi (4y - y^2) y \Delta y$$

$$V = \pi g K \int_0^4 y (1 + \sqrt{y}) (4y - y^2) dy$$

$$t=2 \quad p(t) \cdot K (1 + \sqrt{2-t})$$

$$t=0 \quad \text{distance} = 2-t$$

$$(r=\text{radius})^2 = 4 - t^2$$

$$\cancel{\text{Disk}} \quad \pi g K \int_{-2}^2 (2-t)(1 + \sqrt{2-t})(4-t^2) dt$$

4. [10] Use either the method of Disks or the method of Shells to compute the volume of a torus generated by rotating a circle of radius r about a line R units from the center of the circle.

$$x^2 + y^2 = r^2$$

$$x = \pm \sqrt{r^2 - y^2}$$

$$x = R$$

Disks

$$V_i = \pi [(R + \sqrt{r^2 - y^2})^2 - (R - \sqrt{r^2 - y^2})^2] \Delta y$$

$$= \pi [4R\sqrt{r^2 - y^2}] \Delta y$$

$$V_{\text{disks}} = \pi \int_{-r}^r 4R\sqrt{r^2 - y^2} dy = 4R\pi \cdot \frac{\pi}{2} r^2 = 2\pi^2 R r^2$$

Shells

$$V_i = 2\pi (R-x) (2\sqrt{r^2-x^2}) \Delta x$$

$$= 2\pi \int_{-r}^r 2R\sqrt{r^2-x^2} dx - 2\pi \int_{-r}^r 2x\sqrt{r^2-x^2} dx$$

$$= 2\pi \cdot 2R \cdot \frac{\pi}{2} r^2$$

$$= 2\pi^2 R r^2$$

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Do ONE of the following.

A solid has base bounded by the graphs of $y = \frac{2}{1+x}$, $y = x$, and $x = 0$ with cross sections perpendicular to the x -axis that are square. Find the volume.

OR

Use Integration by Parts to derive the following reduction identity

$$\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx.$$

$$\begin{aligned}
 & \int \cos^n x dx \\
 &= \cos^{n-1} x \sin x + \int (n-1) \cos^{n-2} x \sin^2 x dx \\
 &\quad \text{Note: } \sin^2 x = 1 - \cos^2 x \\
 u &= \cos^{n-1} x \quad dv = \cos x dx \\
 du &= (n-1) \cos^{n-2} x (-\sin x) dx \quad v = \sin x \\
 &= \cos^{n-1} x \sin x + \int (n-1) \cos^{n-2} x dx - (n-1) \int \cos^{n-2} x dx \\
 &\quad \text{Move this to the left side} \\
 n \int \cos^n x dx &= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x dx \\
 \text{so } \int \cos^n x dx &= \frac{\cos^{n-1} x \sin x}{n} + \frac{(n-1)}{n} \int \cos^{n-2} x dx
 \end{aligned}$$