

Math 182
27 Jan 2017

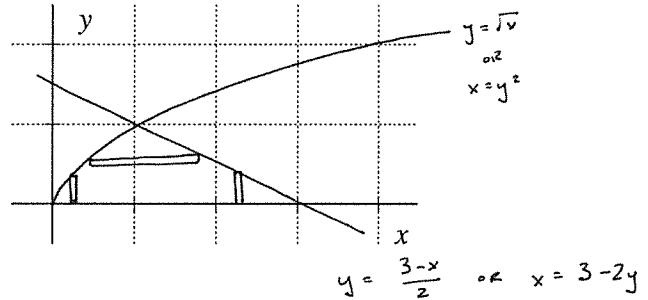
Disks and Shells
Show Appropriate Work

Name: _____
Scaled to 10 points.

1. Consider the region bounded by $y = \sqrt{x}$, $y = (3-x)/2$, and the x -axis. Carefully sketch the region

- (a) 3 The region is rotated about $y = 2$. Express the volume as an integral using the Shell method.

$$V = 2\pi \int_0^1 (2-y) \left[(3-2y) - y^2 \right] dy$$



- (b) 3 The region is rotated about $y = 2$. Express the volume as the sum of two integrals using the Disk method.

$$V = \pi \int_0^1 \left(2^2 - (2-\sqrt{x})^2 \right) dx + \pi \int_1^3 \left(2^2 - \left(2 - \frac{3-x}{2} \right)^2 \right) dx$$

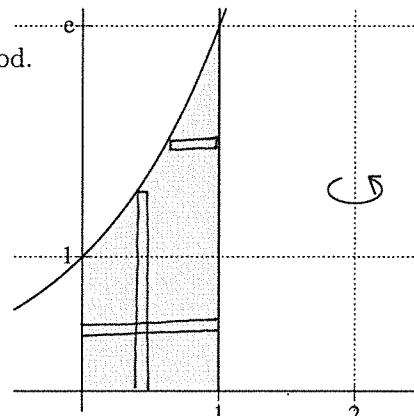
- (c) 6 The region is rotated about $x = 4$. Find the volume.

$$\begin{aligned} V &= \pi \int_0^1 \left[(4-y^2)^2 - (4-(3-2y))^2 \right] dy \\ &= \pi \left(\frac{y^5}{5} - 4y^3 - 2y^2 + 15y \right) \Big|_0^1 \\ &= \pi \left(\frac{1}{5} - 4 - 2 + 15 \right) \\ &= \frac{46\pi}{5} \end{aligned}$$

2. The region in the first quadrant bounded by the graphs of $y = e^x$ and $x = 1$, the shaded region in the figure, is revolved around the line $x = 2$.

3. 3 Express the volume as an integral using the Shell method.

$$V = 2\pi \int_0^1 e^x (2-x) dx$$



4. 3 Express the volume as the sum of two integrals using the Disk method.

$$V = \pi \int_0^1 (2^2 - 1) dy + \pi \int_1^e ((2-\ln y)^2 - 1) dy$$

5. Consider the region bounded by $y = \tan x$ and $y = \sec x$ for $x \in [0, \pi/4]$. Carefully sketch the region. The region is rotated about the line $y = 2$.

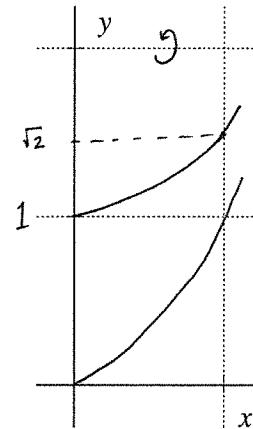
You will find the following identities useful.

$$\tan^2 x + 1 = \sec^2 x$$

$$\int \sec u du = \ln |\sec u + \tan u| + c$$

- (a) [3] Express the volume of the solid using the Disk method.

$$V = \pi \int_0^{\frac{\pi}{4}} [(2 - \tan x)^2 - (2 - \sec x)^2] dx$$



- (b) [3] Express the volume of the solid using the Shell method.

$$V = 2\pi \int_0^1 (2-y) \arctan y dy + 2\pi \int_1^{\sqrt{2}} (2-y) \arcsin y dy$$

$\left(\frac{\pi}{4} \cdot \arcsin y\right)$

- (c) [6] Find the volume.

$$\begin{aligned}
 V &= \pi \int_0^{\frac{\pi}{4}} [4 - 4 \tan x + \tan^2 x - 4 + 4 \sec x - \sec^2 x] dx \\
 &= \pi \int_0^{\frac{\pi}{4}} [4 \sec x - 4 \tan x - 1] dx \\
 &= \pi \left[4 \ln |\sec x + \tan x| - 4 \ln |\sec x| - x \right] \Big|_0^{\frac{\pi}{4}} \\
 &= \pi \left[4 \ln (\sqrt{2} + 1) - 4 \ln \sqrt{2} - \frac{\pi}{4} \right]
 \end{aligned}$$

$\sec^2 x = \tan^2 x + 1$