Use IBP to verify the reduction identity

$$\int \cot^m x \, dx = -\frac{\cot^{m-1} x}{m-1} - \int \cot^{m-2} x \, dx$$

In class we tried

$u = \cot^{m-2} x \, dx$ so $dv = \cot^2 x \, dx$ We have $\cot^2 x = \csc^2 x - 1$ so

$$dv = (\csc^2 x - 1)\, dx$$

giving $v = (-\cot x - x)$ This x caused problems & we got stuck.

The use of the Pythagorean identity $\cot^2 x = \csc^2 x - 1$ is the key to this argument, but it needs to be used <u>before</u> I.B.P.

$$\int \cot^m x \, dx = \int \cot^{m-2} x \left(\csc^2 x - 1\right) dx = \underbrace{\int \cot^{m-2} x \csc^2 x \, dx}_{*} - \int \cot^{m-2} x \, dx \qquad (1)$$

We consider just the first integral $(*)$ & apply IBP to it.

$$\int \cot^{m-2} x \csc^2 x \, dx = -\cot^{m-1} x - (m-2) \int \cot^{m-2} \csc^2 x \, dx$$

$u = \cot^{m-2} x$ $\qquad dv = \csc^2 x \, dx$

$du = (m-2) \cot^{m-3} x \left(-\csc^2 x\right) dx$ $\qquad v = -\cot x$

Applying the standard trick of moving the integral on the right to the left gives

$$(m-1) \int \cot^{m-2} x \csc^2 x \, dx = -\cot^{m-1} x + C.$$

Dividing by $(m-1)$ & substituting back into (1) gives

$$\int \cot^m x \, dx = \frac{-\cot^{m-1} x}{m-1} - \int \cot^{m-2} x \, dx.$$