1. A solid has base bounded by the unit circle $x^2 + y^2 = 1$ and cross sections perpendicular to the $y$-axis are squares. Find the volume of the solid.

\[ V = \pi \int \left( 1 - y^2 \right) dy \]

\[ V = 4 \int_0^1 \left( 1 - y^2 \right) dy = \frac{16}{3} \]

2. The region in the first quadrant bounded by the graphs of $y = e^x$ and $x = 1$, the shaded region in the figure, is revolved around the line $x = 2$.

3. Express the volume as an integral using the Shell method.

\[ V = 2\pi \int_0^1 (2-x)e^x \, dx \]

4. Express the volume as the sum of two integrals using the Disk method.

\[ V = \pi \int_0^1 (4-1) \, dy + \pi \int_1^e \left( 2 - \frac{k}{2} \right)^2 - 1 \, dy \]

5. Find the volume, i.e. evaluate one of the above integrals.

\[ V = 2\pi \int_0^1 (1-x)e^x \, dx = 2\pi \left[ (2-x)e^x \right]_0^1 - \int_0^1 e^y \, dy \]

\[ u = 2-x \quad du = -dx \]

\[ v = e^y \quad dv = e^y \, dy \]

\[ V = 2\pi \left[ e - 2 + e - 1 \right] = 2\pi \left( 2e - 3 \right) \]
6. Express, as an integral, the work (in joules) required to pump all of the water out of the full conical tank in the figure below; water exits through the spout. Distances are in meters, the density of water is \( \rho \), acceleration due to gravity is \( g \). Do not evaluate the integral.

\[
W = \rho g \int_0^4 \left( \frac{y}{4} \right)^2 (5-y) \, dy
\]

7. Integrate

\[
\int x \arctan x \, dx.
\]

\[
u = x, \quad dv = x \, dx
\]

\[
\frac{dv}{dx} = \frac{x^2}{1+x^2}, \quad v = \frac{x^2}{2}
\]

\[
= \frac{x^2}{2} \arctan x - \frac{1}{2} \int \frac{x^2}{1+x^2} \, dx
\]

\[
= \frac{x^2}{2} \arctan x - \frac{1}{2} \left[ x - \arctan x \right] + C
\]