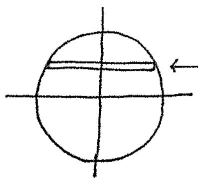
Math 182 | Quiz 2 | Name: ____________
5 Feb 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

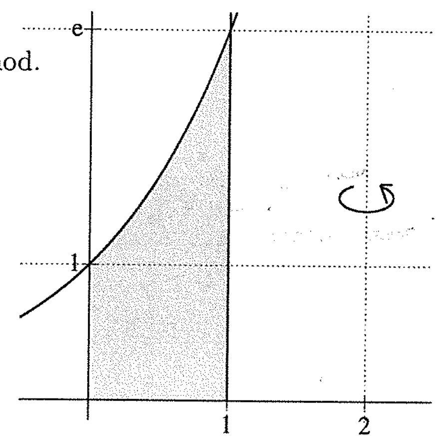
1. $\boxed{3}$ A solid has base bounded by the unit circle $x^2 + y^2 = 1$ and cross sections perpendicular to the y-axis are squares. Find the volume of the solid.



$$V_i = (2x)^2 \Delta y = 4x^2 \Delta y = 4(1-y^2)\Delta y$$

$$V = 4\int_{-1}^{1} (1-y^2)\,dy = 8\left(y - \frac{y^3}{3}\right)\Big|_0^1 = \frac{16}{3}$$

2. The region in the first quadrant bounded by the graphs of $y = e^x$ and $x = 1$, the shaded region in the figure, is revolved around the line $x = 2$.



3. $\boxed{2}$ Express the volume as an integral using the Shell method.

$$V = 2\pi \int_0^1 (2-x)e^x\,dx$$

4. $\boxed{2}$ Express the volume as the sum of two integrals using the Disk method.

$$V = \pi \int_0^1 (4-1)\,dy + \pi \int_1^e \left[(2-\ln y)^2 - 1\right] dy$$

5. $\boxed{3}$ Find the volume, i.e. evaluate one of the above integrals.

$$V = 2\pi \int_0^1 (2-x)e^x\,dx = 2\pi\left[(2-x)e^x\Big|_0^1 + \int_0^1 e^x\right]$$

$u = (2-x)$ $\quad dv = e^x dx$

$du = -dx$ $\quad v = e^x$

$$= 2\pi\left[e - 2 + e - 1\right] = 2\pi(2e-3)$$

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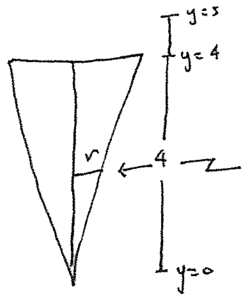
6. $\boxed{5}$ Express, as an integral, the work (in joules) required to pump all of the water out of the full conical tank in the figure below; water exits through the spout. Distances are in meters, the density of water is ρ, acceleration due to gravity is g. **Do not evaluate the integral.**

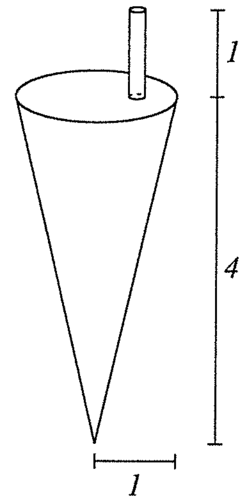


$W_i = \rho g \pi r^2 (5-y)\, \Delta y$

$\frac{r}{y} = \frac{1}{4}$

$r = \frac{y}{4}$

$$W = \rho g \pi \int_0^4 \left(\frac{y}{4}\right)^2 (5-y)\, dy$$



7. $\boxed{5}$ Integrate

$$\int x \arctan x \, dx.$$

$u = \arctan x \qquad dv = x\,dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = \frac{x^2}{2}$

$= \frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$

$= \frac{x^2}{2}\arctan x - \frac{1}{2}\left[x - \arctan x\right] + C$